1 The Number System, LCM & HCF

Direction : In the following questions, select the correct choice among the alternatives given below the question.

1. The expression

$$\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{n(n+1)}\right]$$ for any

natural nuber is

(a) Always greater than 1
(b) Always less than 1
(c) Always equal to 0
(d) Always a negative interger

2. The unit digit in the product of $(2157)^{173}$ is

(a) 4   (b) 0
(c) 6   (d) 5

3. The unit digit in the expression $(11^1+12^2+13^3+14^4 + 15^5 + 16^6)$ is

(a) 1   (b) 9
(c) 7   (d) 0

4. The value is $\sqrt{2\sqrt{2\sqrt{2...\infty}}}$ is

(a) 0   (b) 1
(c) $2\sqrt{2}$ (d) 2

5. If $x=\sqrt{\sqrt{7}+7+\sqrt{(8+2\sqrt{7})}}-\sqrt{7}$, the value of x, will be

(a) $\sqrt{7}-1$ (b) $2\sqrt{7}$
(c) 1   (d) $1-\sqrt{7}$

6. The sum of first 20 odd counting number is

(a) 20   (b) 100
(c) 400   (d) 313

7. The number of zero in 29 is

(a) 4   (b) 6
(c) 3   (d) 8

8. What will be the remainder when (9^6+1) is divided by 8?

(a) 0   (b) 3
(c) 7   (d) 2

9. The digit in the unit place of the number represented by $(7^{95}-3^{58})$ is

(a) 0   (b) 7
(c) 6   (d) 4

10. If $4^{n+1} + x$ and $4^{2n} - x$ are divisible by 5, n being an even integer, the least value of x is

(a) 1   (b) 2
(c) 3   (d) 4

11. The largest natural number by which the product of three consecutive even natural numbers is always divisible by is

(a) 16   (b) 24
(c) 48   (d) 96

12. What smallest number of six digits is divisible by 111 ?

(a) 111111   (b) 110011
(c) 100011   (d) none of these

13. Which of the following number is exactly divisible by all prime numbers between 1 and 17 ?

(a) 515513   (b) 440440
(c) 345345   (d) 510510

14. On dividing a certain number by 5, 7 and 8 successively, the remainders are 2, 3 and 4 respectively.

What would be the temainders if the order of the division is reversed?

(a) 3, 2, 1   (b) 4, 4, 3

(c) 5, 5, 2   (d) 4, 3, 1

15. If $-1 \leq y \geq 3$, the least possible valued of $(2y - 3x)$ is

(a) 0   (b) -3

(c) -4   (d) -5

16. Which among the following is greatest $\sqrt{5}, \sqrt[3]{11}, \sqrt[6]{123}$?

(a) $\sqrt{5}$,   (b) $\sqrt[3]{11}$

(c) $\sqrt[6]{123}$ (d) All are equal

17. Which among the following is greates $\sqrt{7} + \sqrt{3}, \sqrt{5} + \sqrt{5}, \sqrt{6} + 2$?

(a) $\sqrt{7} + \sqrt{3}$,   (b) $\sqrt{5} + \sqrt{5}$

(c) $\sqrt{6} + 2$ (d) All are equal

18. Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54.

(a) 10244   (b) 10296

(c) 10368   (d) 10291

19. The LCM of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{9}{13}$ is

(a) 36   (b) $\frac{1}{36}$

(c) $\frac{1}{1365}$ (d) $\frac{12}{455}$

20. The LCM of $(16-x^2)$ and (x^2+x-6) is

(a) $(x-3)(x+3)(4-x^2)$

(b) $4(4-x^2)(x+3)$

(c) $4(4-x^2)(x-3)$

(d) $(16-x^2)(x-23)(x+3)$

21. GCD of $(x^2 -4)$ and $(x^2 +x-6)$ is

(a) $(x+2)$ (b) $(x-2)$

(c) (x^2-2) (d) (x^2+2)

22. The HCF of two numbers is 12 and their difference is 12, The numbers are

(a) 66, 78   (b) 70, 82

(c) 94, 106   (d) 84, 96

23. The cube root of 1.061208 is

(a) 1,022   (b) 10.22

(c) 0.102   (d) 1.02

24. The sum of the digits of a 3 digit number is subtracted from the number. The resulting number is always?

(a) divisible by 6

(b) not divisible by 6

(c) divisible by 9

(d) not divisible by 9

25. Rohit, Harsha and Sanjeev are three typists, who working simultaneously can type 216 pages in four hours. In on hour, Sanjeev can type as many pages more than Harsha as Harsha can type more than Rohit. During a period of five hours, Sanjeev can type as type as many pages as Rohit can during seven hours.

How many pages does each of them type per hour respectively?

(a) 14, 17, 20   (b) 16, 18, 22

(c) 15, 17, 22   (d) 15, 18, 21

26. If the numbers from to 45 which are exactly divisible by 3 are arranged in acending order, minimum number being on the top, which would come at the nintth place from the top?

(a) 18   (b) 24

(c) 21   (d) 27

27. In a objective examination of 90 questions, 5 marks are allotted for every correct answer and 2 marks are deducted

for every wrong answer. After attempting all the 90 questions a student got a total of 387 marks. Find the number of questions that he attempted wrong.

(a) 32

(b) 25

(c) 29

(d) 49

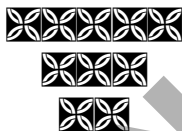28. If m and n are natural number such that $2^m - 2^n = 960$, what is the value of m?

(a) 10

(b) 12

(c) 16

(d) can't determine

29. A number when divided by 765 leaves a reminder 42. What will be the remainder if the number is divied by 17?

(a) 8

(b) 7

(c) 6

(d) 5

30. If 11,109,999 is duvuded by 111, then what is the remainder?

(a) 1098

(b) 11888

(c) 1010

(d) 1110

Solution

1. **(b)** $\left[\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+...+\dfrac{1}{n(n+1)}\right]$

$$=\left(1-\dfrac{1}{2}\right)+\left(\dfrac{1}{2}-\dfrac{1}{3}\right)+\left(\dfrac{1}{3}-\dfrac{1}{4}\right)+\left(\dfrac{1}{n}-\dfrac{1}{n+1}\right)$$

$$=1-\dfrac{1}{2}+\dfrac{1}{2}-\dfrac{1}{3}+\dfrac{1}{3}-\dfrac{1}{4}+...$$

$$+\dfrac{1}{n}-\dfrac{1}{n+1}=1-\dfrac{1}{(n+1)}=\dfrac{n}{n+1}$$

2. **(b)** Unit digit in the product of any number with 5 at unit place is always 5.

∴ unit digit in $(25)^{6251}$ is 5

Similarly, unit digit in the product of any number with 6 at unit place is always 6.

unit digit in $(36)^{528}$ is 6

Now unit digit in $(3)^4$ is 1

∴ unit digit for the expression

$$\left[(25)^{6251}+(36)^{528}+(73)^{54}\right]$$

$$=5+6+9=20\Rightarrow 0$$

3. **(b)** unit digit in $(1)^1=1,(2)^2=4$, $(3)^3=7,(4)^4=6,(5)^5=5$, $(6)^6=6$

$$=1+4+7+6+5+6=29$$

$$\Rightarrow 9$$

4. **(d)** Let $a=\sqrt{2\sqrt{2\sqrt{2...\infty}}}$

$$\Rightarrow a=\sqrt{2(a)}$$
$$\Rightarrow a2=\left(\sqrt{2a}\right)^2=2(a)$$

∴ a is either 0 or 2 Since, a cannot be zero.

Hence a = 2

5. **(c)** $x=\sqrt{\sqrt{7}+7+\sqrt{(8+2\sqrt{7})}}-\sqrt{7}$

$$8+2\sqrt{7}=7+1+2\sqrt{7}=\left(\sqrt{7}+1\right)^2$$

Therefore

$$x=\sqrt{\sqrt{7}+7+\left(\sqrt{7}+1\right)}-\sqrt{7}$$

$$=\sqrt{8+2\sqrt{7}}-\sqrt{7}$$
$$=\left(\sqrt{7}+1\right)-\sqrt{7}$$

$$=1$$

6. **(c)** Sum of AP of first 20 odd numbers $1+3+5+7$ .... is given by

$$\dfrac{n}{2}\left[2a+(n-1)d\right]$$
$$=\dfrac{20}{2}\left[2\times 1+(20-1)2\right]=400$$

7. **(c)** Number of zero in 29

$$=\dfrac{29}{5}+\dfrac{29}{5^2}=5+1=6$$

∴ Number of zero is 29=6

8. **(d)** Usisng the remainder theorem the following expressions will have the same remainder.

$$\dfrac{9^6+1}{8}=\dfrac{1^6+1}{8}\Rightarrow\dfrac{2}{8}\Rightarrow$$

Remainder is 2.

9. **(d)** Unit digit in 7^4 is 1.

∴ Unit digit in 7^{92} is 1

∴ Unit digit in 7^{95} is 3

Unit digit in 3^4 is 1

∴ Unit digit in 3^{56} is 1

hence Unit digit in 3^{58} is 9

Now unit digit in $(7^{95}-3^{58})$ is $(3-9) = 4$ [$\because$ 13 -9 = 4]

10. **(a)** For $(4^{n+1}+x)$ and $(4^{2n} - x)$ is n is taken 2, then for x = 1

11. **(c)** The required number is 2x4x6 = 48

12. **(c)** The smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder

Since remainder is more than half of 111.

Hence number to be added to make it divisible is (111-100)=11

$\therefore$ Required number= 100011

13. **(d)** Prime number between 1 and 17 are 2, 3, 5, 7, 11 and 13. Now we find that number 5120510 is divisible by each of these prime numbers.

14. **(c)** N=5 [7 (8x+4) + 3] + 2

= 35 (8x)+ 157 = 280x +157

Now if the number 280x + 157 is divided by 8.

Quotient= 35x+19 and remainder = 5

When (35x+19) is divided by 7

Quotient= 5x+2 and remainder = 5

When (5x+2) is divided by 5

Quotient = x and remainder = 2

Hence respective remainders are 5, 5, 2.

15. **(c)** For (2x - 3x) to be minimum, take the least value of y and greateds value of x.

[x = -1, 0, 1, 2 and y = 1, 2, 3]

$\therefore$ Required value

= $(2 \times 1 - 3 \times 2) = -4$

16. **(a)** $(5)^{1/2}, (11)^{1/3}, (123)^{1/6}$

$\Rightarrow (5^3)^{1/6}, (11^2)^{1/6}, (123)^{1/6}$

$\Rightarrow (125)^{1/6}, (121)^{1/6}, (123)^{1/6}$

Among these three numbers $(125)^{1/6}$ is the greatest. Hence, $\sqrt{5}$ is our answer.

17. **(b)** $\sqrt{7}+\sqrt{3}, \sqrt{5}+\sqrt{5}, \sqrt{6}+2$

Squaring these numbers

$7 + 3 + 2\sqrt{21}$, $5+5+2\sqrt{25}$

$6 + 4 + 4\sqrt{6}$

Clearly, $10 + 2\sqrt{25}$ is the greatest.

18. **(c)** Smallest number of five digits = 10000

LCM of 16, 24, 36, 54 = 432

If we divide 10000 by 432, we get a remainder of 64,

$\therefore$ Required value

= 10000+(432-64) = 10368

19. **(c)** LCM of $\dfrac{2}{3}, \dfrac{3}{5}, \dfrac{4}{7}, \dfrac{9}{13}$

$= \dfrac{\text{LCM of}(2,3,4,9)}{\text{HCF of}(3,5,7,13)} = \dfrac{36}{1} = 36$

20. **(d)** $(16-x^2) = (4-x)(4+x)$

$(x^2+x-6) = (x+3)(x-2)$

LCM $= (16-x^2)(x-2)(x+3)$

21. **(b)** $(x^2-4) = (x-2)(x+2)$

$(x^2+x-6) = (x-2)(x+3)$

GCD or HCF = (x - 2)

22. **(d)** Out of the four alternatiives (84, 96) is the pair of numbers with

HCF 12 and difference 12.

23. (d) $\sqrt[3]{1.061208} = 1.02$

24. (c) Let the three digit

= (4+3+9) = 16

Clearkt 439-16 = 423 is divisible by 9.

25. (d) Let Rohit's typing in 7 be x pages.

Then, his 1 h typing - $\dfrac{x}{7}$ pages.

Sanjeev's typing in 5h = $\dfrac{x}{5}$ pages

∴ Harsha's typing in 1 h

$= \left(\dfrac{x}{5} + \dfrac{x}{7}\right) \times \dfrac{1}{2} = \dfrac{6x}{35}$

∴ $4\left(\dfrac{18x}{35}\right) = 216 = x = \dfrac{216 \times 35}{18 \times 4} = 105$

∴ Required ratio

$= \dfrac{105}{7} : \dfrac{105}{5} : 105 \times \dfrac{6}{35}$

= 15 : 18 : 21

26. (d)

27. (c) Let the wrong questions be x. We get the equations

$(90-x) \times 5 - x \times 2 = 387$
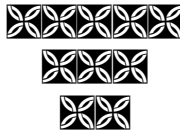
$\Rightarrow x = 9$

28. (a) Work from the choices

$2^{10} - 2^6 = 1024 - 64 = 960$

29. (a) (765K + 42)/7. Since 765 is divisible by 17 the remainder will be $\dfrac{42}{17} = 8$

30. (d)

The word percentage means per hundred. For instance, if a person saves 20% of his salary, he is said to save 20 parts out of 100 parts.

Percentage

1. For two successive increments/ Discount

$$= x + y + \frac{xy}{100}$$

2. If A > B by x%, Then B < A by

$$= \left(\frac{x}{100 + x} \right) \times 100$$

3. If A < B by x% Then B > A by

$$= \left(\frac{x}{100 - x} \right) \times 100$$

4. Population of a town after 'n' years

$$= P \left(1 + \frac{R}{100} \right)^n$$

P= Present Population

R= Rate %

5. Population of a town before 'n' years

$$= P \left(1 + \frac{R}{100} \right)^n$$

7. Value of Machine before 'n' years

$$= \frac{P}{\left(1 - \frac{R}{100} \right)^n}$$

8. First year population R1%   ↑

   Second year →   R2%   ↑

   Third year   →   R2%   ↑

   After 3 years

$$= P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_1}{100} \right)$$

9. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

10. Originol Price

$$= \frac{Price \times Reduced\%}{Quantity \times (100 - R\%)}$$

11. Valume Increment of any polygon

$$= x + y + z + \frac{xy}{100} + \frac{yz}{100} + \frac{xy}{100}$$

$$\frac{+xyz}{10,000}$$

Example 1: Quantity of water in milk constitutes 6 parts of every 20 parts of mixture. What is the percentage of water in the mixture?

Sulution: Percentage of water in the mixture $= \frac{6}{20} \times 100 = 30\%$

To convert any fraction a/b to rate precentage, multiply it by 100 and put % sign. Alternatively to convert a rate percent to a fraction, divide it by 100 and delete the % sign.

Percentage Increase/ Percentage Decrease: Increase or decrease is always in absolute terms whereas percentage increase/ decrease is expressed in percentage terms. Percentage increase / decrease is calculated wityh respect

to the base (Previous) value unless mentioned otherwise.

Example 2 : The production of a company is increased from 150 crores to 350 crores in a financial year 2001-2002 and is decreased by 50 crore in the subsequent year. What is percentage increase and decrease in production in the respective years?

Solution: Increase in production in 2001-2002

$= (350 - 150) = 200$ crores.

% increase in production

$$=\frac{\text{Increase}}{\text{BaseValue}}\times100=\frac{200}{150}\times100$$

$$=133\frac{1}{3}\%$$

Decrease in production in 2002-2003 = 50 crores % decrease in production during 2002-2003.

$$=\frac{\text{Decrease}}{\text{BaseValue}}\times100$$
$$=\frac{50}{350}\times100=\frac{100}{7}=14\frac{2}{7}\%$$

Percentage increase / decrease

$$=\frac{\text{Increase}/\text{Decrease}}{\text{BaseValue}}\times100$$

It is important to note here that the base value for the next year is 350 crores and not 150 crores.

Example 3: If A's income is 20% more than that of B, then how much percent is B's income less than that of A?

Solution: Let the income of B be Rs. 100 then income of A=Rs. 120

In the question B's income is being

compared with that of A snd hence base value to find the % decrease will be the income of A.

$$\% \text{ decrease } =\frac{\text{Decrease}}{\text{BaseValue}}\times100$$

$$=\frac{(120-100)}{120}\times100=\frac{20}{120}\times100$$
$$=\frac{50}{3}\% \text{ or} 16\frac{2}{3}\%$$

Example 4 : The price of a product is increased by 20%. If the original price is Rs. 300. What is the final price of the product?

Solution: Final price = Initial price + Initial price $\times\dfrac{20}{100}$

$$= \text{Initail price}\left(1+\frac{20}{100}\right)=\text{Initial price}\left(\frac{120}{100}\right)$$

$= $ Initial price $\times$ 1.2

$= 300 \times 1.2 =$ Rs. 360

(a) If a quantity is increase by r%, then final quantity (after increase) is obtained by multiplying the original quantity by $\left(\dfrac{100+r}{100}\right)$

(b) Likewise, if a quantity is decrease by r% the final quantity (after decrease) is obtained by multiplying the original quantity by $\left(\dfrac{100-r}{100}\right)$

Example 5 : If the price of petrol is incrased by 20% and subsequently by 40%. What is final price per litre, if the original price was Rs. 25 per litre?

Solution: Final Price = $25\times 1.2 \times 1.4$

$$= 25 \times \frac{6}{5} \times \frac{7}{5}$$

= Rs. 42

Example 6 : The wages of a worker is first increased by 20% and subsequently reduced by 20%. If the original monthly wages is Rs. 500. What is the final wages?

Solution: Final Price = 500 × 1.2 × 0.8

$$= 500 \times \frac{6}{5} \times \frac{4}{5} = Rs.480$$

Successive Increase/ decrease : Examples 5 and 6 show the successive application of the percentage to a given (original) value. All successive changes in % (increase or decrease) can be represented as a single percentage.

Whish is given by $\left[a + b + \frac{ab}{100} \right]\%$. Where a and b show the first and second percentage changes.

Now, with the help of the above rule a single % change in example 6 can be calculated as

$$\left[(+20) + (-20) + \frac{(-20) \times (+20)}{100} \right]\%$$

= -4%

∴ Final wages = 500-20=Rs. 480
[4% of 500 is 20]

Here + ve sign shows the increase and negative sign shows the decrease.

Example 7 : If the price of petrol increases successively by 20% and then by 10%, what is the net change in percentage terms?

Solution: Net change in %

$$= \left[20 + 10 + \frac{200}{100} \right]\% = 32\%$$

It means tyhat the successive increase of 20% and 10% are equal to a single increase of 32%

Example 8 : If the price of a commodity is increased by 20% find by how much percent must a housewife reduce her consumption so as not to invrease the expenditure?

Solution: So in order to keep the expenditure fixed, it is essential that:

-      If the rate of the item increases, then consumption will increase.

-      If the rate of the item decreases, then consumption will increase.

% Change in consumption

$$= \frac{\% \, change \, in \, rate}{100 + \% \, change \, in \, rate} \times 100$$

Therefore, for the example 8,
% reduction in consumption

$$= \frac{20}{100 + 20} \times 100 = \frac{20}{120} \times 100$$

$$= \frac{50}{3}\%$$

Example 9 : The length of a rectangle is increased by 20% and breadth is decreased by 10%. Calculate the percentage change in the area.

Solution: Percentage change in area

$$= a + b + \frac{ab}{100}$$

$$= 20 - 10 - \frac{200}{100} = 8\%$$

Example 10 : After two successive decrease of 20%, the price of television is Rs.

12,800. What is the original price?

Solution: Let original price be x,

then $\quad x \times 0.8 \times 0.8 = 12,800$

$\Rightarrow \quad x \times \dfrac{4}{5} \times \dfrac{4}{5} = 12,800$

$\therefore \quad x = Rs.\ 20,000$

Example 11: The radius of a circle has increased by 20%. By what percentage does.]

(a) the circumference increase?

(b) the area increase?

Solution: (a) Circumference of a circle = 2n r or circumference is directly proportional to radius or circumference α (r).

Hence, if radius is increased by 20%, the circumference also increases by 20%

(b) Area of circle n r² or Area of circle is directly proportional to (r)2 or Area α (r)²

Therefore, two successive increase of

$20\% = 20 + 20 + \dfrac{20 \times 20}{100}$

$= 44\%$

Example 12: In an examination 35% of total students faild in Hindi, 45% failed in English and 20% in both. Find the percentage of those who passed in both in the subjects.
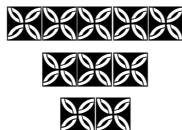

n (B) failed in english = 45   n (A) failed in Hindi = 35

n(A ∩ B) Failed in both = 20
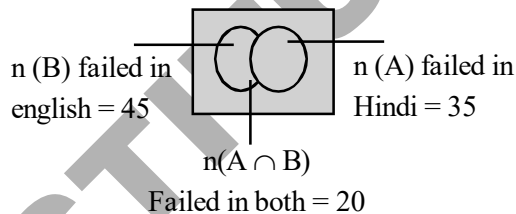
Solution: We know that n (A ∪ B) = n (A) + n (B) - n (A ∩ B)

$= (35 + 45 - 20) = 60$

$\therefore$ n (A ∪ B) = number of failed in one or other or both = 60

$\therefore$ Number of passed = (100 - 60) = 40

Exercise

1. Two successive discounts of 8% and 12% are equal to a single discount of

   (a) 20%   (b) 19.04%

   (c) 20.96%   (d) 22%

2. The population of a city is estimated to be 4,32,000 after 2 years. If the population growth is 20% per annum. What is the current population?

   (a) 3,10,000   (b) 2,85,000

   (c) 3,80,000   (d) 3,00,000

3. The price of a product of a company increases by 10% and the turnover increases by 10%. What is the change in quantity sold?

   (a) 10%   (b) 8%

   (c) 20%   (d) No change

4. The price of a television includes the manufacturing cost, 10% sales tax and 10% profit. What is the manufacturing cost, If the price is Rs. 14,400? (Sales tax and profit are to be calculated on manufacturing cost.)

   (a) 10,000   (b) 12,000

   (c) 12,500   (d) 9,000

5. A class has girls and boys in the ratio 4 : 5. Among the girls, the ratio of mathematics to physics students on the entire class is 3 : 2.

   What percentage of class comprises girls studying mathematics?

   (a) 33.3%   (b) 30%

   (c) 25%   (d) 18%

6. Asha's income is Rs. 12,000 per month. She pays 20% tax on monthly income above Rs. 2,000 and she spends 30% of the remaining income.

   How much does she save annually?

   (a) 70,000   (b) 60,000

   (c) 84,000   (d) 50,000

7. If 10% of an electricity bill is deducted, Rs. 45 is still to be paid. How much was the bill?

   (a) Rs. 50   (b) Rs. 40

   (c) Rs. 35   (d) Rs. 54

8. A reduction of Rs. 2 per kg enables a man to purchase 4 kg more sugar for Rs. 16. Find the original price of sugar.

   (a) Rs. 4 per kg   (b) Rs. 6 per kg

   (c) Rs. 3 per kg   (d) Rs. 4.5 per kg

9. If the length of a rectangle is decreased by 60% and the width is increased by 30%, then what will be the % change in the area of the rectangle?

   (a) 30% decrease   (b) 45% decrease

   (c) 48% decrease   (d) 48% Increase

10. Anand spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. His savings are increased by.

    (a) 10%   (b) 25%

    (c) $37\frac{1}{2}\%$ (d) 50%

11. In an examination, 35% candidates failed in one subject and 42% failed in another subject while 15% failed in both the subjects. If 2500 candidates appeared at the examination. How many passed in either subject but not in both?

(a) 325   (b) 1175

(c) 2125   (d) none of these

12. On increasing the price of T.V. sets by 30%, their sale decreases by 20%. What is the effect on the revenue receipts of the shop?

(a) 4% Increase (b) 5% decrease

(c) 8% Increase (d) 8% decrease

13. The price of oil is increased by 25%. If the expenditure is not allowed to increase, the ratio between the reduction in consumption and the original consumption is

(a) 1 : 3   (b) 1 : 4

(c) 1 : 5   (d) 1 : 6

14. A quantity of 30 ml of 20% alcohal is mixed with 20 ml of 25% alcohol. What is the strength of alcohol in the mixture?

(a) 20%   (b) 25%

(c) 22%   (d) 22.5%

15. A fruit vendor gives two successive discounts of 10% and 14%. What is the overall discount given?

(a) 22.4%   (b) 22.6%

(c) 23.4%   (d) 23.6%

16. Sunder sell his goods 20% cheaper than Kundan and 20% dearer than Aditya's goods cheaper/ dearer than Kundan?

(a) 33.33%   (b) 50%

(c) 42.85%   (d) none of these

17. An inspector rejects 0.08% of the metres as defective. How many will he examine to reject 2?

(a) 200   (b) 250

(c) 2500   (d) 3000

18. If 50% of the 2 : 3 solution of milk and water is replaced with water, then the concentration of the solution is reduced by?

(a) 25%   (b) 33.33%

(c) 50%   (d) 75%

19. Sunder purchased an office bag with a price tag of Rs. 600 in a sale where 25% discount was being offered on the tag price.

He was given a further discount of 10% on the amount arrived at after giving usual 25% discount. What was the final amount paid by Sunder?

(a) Rs. 210   (b) Rs. 540

(c) Rs. 405   (d) Rs. 450

20. A cycle agent buys 30 bicycles of which 8 are first grade and the rest are second grade for Rs. 3150. Find at what price he must sell the first grade bicycles so that if he sells the second grade bicycles at three quarters of the price, he may makea profitof40% on his outlay?

(a) Rs. 200   (b) Rs. 240

(c) Rs. 180   (d) Rs. 210

21. P% of a number P is q% more than r% of the number R. If the difference between

P and R is r% or R and if the sum of P and R, is 210, then which of the following statements is always true?

(a) 1 : 15   (b) 1 : 10

(c) 1 : 20   (d) 1 : 12

22. A bicycle originally costs Rs. 100 and was discounted 10% After three months it was sold after being discounted 15%. How much was the bicycle sold for?

(a) 55.5   (b) 95.25

(c) 76.5

(d) none of the above

23. If the height of a triangle is decreased by 40% and its base is increased by 40%, What will be the effect on its area?

(a) No change   (b) 8% decrease

(c) 16% decrease   (d) 16% Increase

24. Fresh grapes contain 80 percent water while dry grapes contain 10 [ercent water. If the weight of dry graped is 250 kg what was its total weight when it was freash?

(a) 1000kg   (b) 1125kg

(c) 1225kg   (d) 1100kg

25. Due to global recession starting in January, Ram's monthly salary of Rs. 80,000 was cut by 10%. The monthly expenses, which were Rs. 6000, incrased at the rate of 5 per month. Since which month will he have no savings if the recession lasted for a year?

(a) April   (b) March

(c) May   (d) June

26. Ravi's salary is 150% of Amit's salary. Amit's salary is 80% of Ram's salary. What is the ratio of Ram's salary to Ravi's salary?

(a) 1 to 2   (b) 2 to 3

(c) 5 to 6   (d) 6 to 5

27. The length of a rectangular plot is increased by 25% to keep its area unchanged, the width of the plot should be.

(a) kept unchanged

(b) Increased by 25%

(c) Increased by 20%

(d) Reduced by 20%

28. the population of bacteria culture doubles every 2 min. Approximately, how many minutes will it take for the population to grow from 1000 to 500,000 bacteria?

(a) 10   (b) 12

(c) 14   (d) 18

29. A sink contains exactly 12 L. of water. If water is drained from the sink until it holds exactly 6 L. of water less than the quantity drained away, how many liters of water were drained away?

(a) 2   (b) 6

(c) 3   (d) 9

30. A chemist has 10 L. of a solution that is 10 percent nitric acid by volume. He wants to dilute the solution to 4 percent strength by adding water. How many liters of water must be add?

(a) 5   (b) 20

(c) 18   (d) 25

Solution

1. (b) Single discount $= \left(a + b + \dfrac{ab}{100}\right)\%$

$$= \left[-8 - 12 + \frac{(-8) \times (-12)}{100}\right]\%$$

$= [-20 + 0.96]\% = -19.04\%$

(-ve) sign is used because discount refers to the decrease in values.

2. (d) Single percentage change.

$$\left(20 + 20 + \frac{20 \times 20}{100}\right)\% = 44\%$$

Population after 2 years = Present population $\times 1.44$

$\therefore$ Present population $= \dfrac{4,32,000}{1.44}$

$= 3,00,000$

3. (d) Let the price per piece and quantity in piece be Rs. 10 and 10 respectively.

then original turnover $= 10 \times 10 = 100$

[$\therefore$ Turnover = Price $\times$ Quantity)

Now new turnover = 110 and new price = Rs. 11.

The quantity $= \dfrac{\text{Turnover}}{\text{Price}} = \dfrac{110}{11} = 10$

Hence, there is no change in the quantity sold.

4. (b) Since sales tax and profit are to be calculated on manufacturing cost, therefore single percentage change = 20%

Now, manufacturing cost $\times 1.2 = 14,00$

$\therefore$ Manufacturing cost $= \dfrac{14,400}{1.2}$

$= $ Rs. 12,000

5. (a) Let girls and body in the class 4x and 5x respectively

$\therefore$ Total students $= 4x + 5x = 9x$

Girls students of Mathematics

$= \dfrac{3}{4} \times 4x = x$

Girls students of Physics $= \dfrac{1}{4} \times 4x = x$

Percentage of Girls students of Mathematics $= \dfrac{3x}{9x} \times 100 = 33.3\%$

6. (c) Tax paid $= (12000 - 2000) \times \dfrac{20}{100}$

$= $ Rs. 2000

Amount after tax $= (12000 - 2000) = $ Rs. 10,000

Other expenses $= 10000 \times \dfrac{30}{100}$

$= $ Rs. 3000

Saving $= 10000 = 3000 = $ Rs. 7000

Annual saving $= 12 \times 7000 = $ Rs. 84000

7. (a) Let the bill be Rs x.

Then, $x \times 0.9 = 45$

$\therefore \qquad x = $ Rs. 50

8. (a) Let the original price of sugar be Rs. x per kg.

Then, quantity purchased for

Rs. $16 = \dfrac{16}{x}$ kg

After reduction of Rs. 2 per kg. quantity purchased.

for Rs. $16 = \dfrac{16}{(x-2)}$ kg

From the given information,

$\dfrac{16}{(x-2)} = \dfrac{16}{x} + 4 \Rightarrow \dfrac{16}{(x-2)} - \dfrac{16}{x} = 4$

or $4\left[\dfrac{1}{(x-2)} - \dfrac{1}{x}\right] = 1 \Rightarrow 4\left[\dfrac{x - x + 2}{x^2 - 2x}\right] = 1$

or $x^2 - 2x - 8 = 0$

or $x = 4$, $x = -2$

ignoring the (-he) value, $x = 4$

9. (c) Area = length $\times$ width

single % change in area

$$= \left[-60 + 30 + \frac{(-60) \times (30)}{100} \right]\%$$

$= -30 - 18 = -48\%$

$\therefore$ Area will decrease by 48%

10. (d) Let the income of Anand be Rs. 100.

He spends = Rs.75, Saving = Rs. 25

Income after increase = Rs. 120

Expenses after increase = Rs. 82.50

Saving = 37.50

Increase in saving   = 37.50 - 25

= Rs. 12.50

Increase % $= \dfrac{\text{Increase}}{\text{Base value}} \times 100$

$$= \frac{12.50}{25} \times 100 = 50\%$$

11. (b) Failed in first subject

$$= 2500 \times \frac{35}{100} = 875$$

Failed in second subject

$$= 2500 \times \frac{42}{100} = 1050$$

Failed in both the subject

$$= 2500 \times \frac{15}{100} = 375$$

Failed in first subject only

= Passed in second subject only

= (875 - 375) = 500

Failed in second subject only

= Passed in first subject only

= (1050 - 375) = 675

$\therefore$ Passed in either subjects but not in both = (500 + 675) = 1175

12. (a) Ravenue = Price $\times$ Sale

Single percentage change

$$= \left[30 - 20 + \frac{(30) \times (-20)}{100} \right]\%$$

$= 10 - 6 = 4\%$

$\therefore$ The revenue will increase by 4%

13. (c) Let the original price of oil be Rs. x per litre.

After increase the price of oil will be Rs. 1.25x per litre.

Since the expenditure has to be same even after increase i.e. Rs. x

Therefore, quantity of oil bought for Rs. x after increase $= \dfrac{1}{1.25x} \times x = \dfrac{4}{5}$ litre

Reduction in consumption

$$= \left(1 - \frac{4}{5} \right) = \frac{1}{5} L$$

Reduction in consumption : Priginal consumption $= \dfrac{1}{5} : 1 = 1 : 5$

14. (c) Quantity of alcohol in 30 ml. solution

$= 30 \times 20\% = 6$ ml.

Quantity of alcohol in 20 ml. solution

$= 20 \times 25\% = 5$ ml.

Percentage of alcohol in the mixture

$$= \frac{6 + 5}{(30 + 20)} \times 100 = 22\%$$

15. (b) Single percentage change

$$\left(a + b + \frac{ab}{100} \right)\%$$

$$= \left(-10 - 14 + \frac{(-10) \times (-14)}{100} \right)\%$$

$$= (-24 + 1.4)\% = -22.6\%$$

16. **(a)** Let Sunder sells his good for Rs. 100

Kundan sells his goods for

$$\frac{100}{0.8} = Rs.\,125$$

and Aditya sells his goods for $\dfrac{100}{1.2}$

= Rs. 83.33

Aditya goods is (125-83.33)

cheaper than Kundan's

% cheaper

$$= \frac{41.67}{125} \times 100 = 33.33\%$$

17. **(c)** Let the inspector examined x metres, then 0.08% of x = 2

$$\Rightarrow \quad \frac{x \times 0.08}{100} = 2$$

$$\Rightarrow \quad x = \frac{200}{0.08} = 2500$$

18. **(c)** Let the quantity of milk and water be 40 and 60 respectively, after removing 50% of solution.

quantity of milk = 20 and quantity of water = 30

Therefore, the concentration of the solution is reduced from 40 to 20 i.e., reduced by 50%.

19. **(c)** Final amount after giving successive discounts of 25% and 10%.

$$= 600 \times 0.75 \times 0.9 = Rs.\,405$$

20. **(c)** 8x + 2y = 3150

Let he sells A grade cycle at a rate of Rs. x per bicycle, then

$$8x + \frac{22 \times 3}{4} x = 3150 \times 1.4$$

32x + 66x = 17640

98x = 17640

$$\Rightarrow \qquad x = 180$$

∴ He should sell the first grade bicycle at a rate of Rs. 180.

21. **(b)** Let the quantity of milk purchased be x and quantity of water added be y. Then the ratio of water to milk be y : x. Then, the ratio of water to milk be y : x.

CP = 6.4x, SP = 8 (x+y), Profit % = 37.5%

8 (x+y) = 6.4x × 1.375

8x + 8y = 8.8x

$$\Rightarrow \qquad 8y = 0.8x$$

$$\Rightarrow \qquad \frac{x}{y} = \frac{80}{8} = \frac{10}{1}$$

$$\Rightarrow \qquad y : x = 1 : 10$$

22. **(c)** SP of bicycle = 100 × 0.9 × 0.85

= Rs. 76.50

23. **(c)** Let the height of the triangle be 'h' and the base be 'b' area $= \dfrac{1}{2} bh$

Decreased height $= \dfrac{6}{10} h$

Increased base $= \dfrac{14}{10} b$

Area $= \dfrac{1}{2} \times \dfrac{6}{10} h \times \dfrac{14}{10} b$

$$= \frac{1}{2} \times \frac{84}{100} bh$$

Decreased in area $= \dfrac{1}{2} bh \left(1 - \dfrac{84}{100} \right)$

Percentage decrease

$$= \frac{1}{2} bh \times \frac{16}{100} \times \frac{2}{bh} \times 100 = 16\%$$

24. **(b)** In 250 kg, there is 225 grape powder and 25 water. Now 225=20%, Since 80% is water in fresh grapes. So total weight

of freash grapes = $225 \times 5 = 1125$.

25. (a)

26. (c) Let the salary of Ram be Rs. 100, then salary of Amit

= Rs. 80 and salary of Ravi= Rs. 120

∴ Ratio of Ram's salary to Ravi's salary

= 100 : 120 = 5 : 6

27. (d) Let the original length and breadth be 10 cm each then original area = 100cm²

New length = $10 \times 1.25 = 12.5$ cm

let new breadth be x.

Then, $12.5x = 100$

⇒ $x = \dfrac{100}{12.5} = 8cm$

28. (d) In 2 min it becomes 2000

In 4 min it becomes 4000

In 6 min it becomes 8000

Likewise in 18 min it becomes 512000

29. (d) Water (W) + drained (D) = 12

⇒   W+D=12

Now, W=D - 6

⇒   D-6 + D = 12

⇒D=9

30. (a) Quantity of nitric acid $= 10 \times \dfrac{1}{10} = 1L$

Water = 10-1 = 9L

Let x litre of water be added.

Then,

$(10+x) \times \dfrac{4}{100} = 1$ ⇒ x = 5
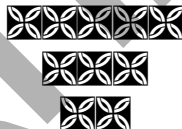
The price at which a person buys a product is called the cost price of the product and the price at which a person sells a product is called the selling price of the product. In a transaction if the selling price is more than the cost price, it gives a profit or in other words.

Profit = (selling price-cost price). On the other hand, if the selling price is less than the cost price, the transaction results into loss and hence loss = (cost price - selling price). It should be noted that the selling price of the seller is the cost price of the buyer. Therefore, in case of profit the following formula are applicable:

$$\text{Profit} = SP - CP$$

$$\text{Profit percentage} = \frac{\text{Profit}}{CP} \times 100$$

Profit percentage is always calculated on CP unless specified.

Profit/Loss

1. C.P. $\rightarrow$ Cost Price
2. S.P. $\rightarrow$ Selling Price
3. Profit = S.P. - C.P.
4. Loss = C.P. - S.P.
5. Profit % $= \dfrac{\text{Profit}}{\text{C.P.}} \times 100$
6. Loss % $= \dfrac{\text{Loss}}{\text{C.P.}} \times 100$
7. S.P. $= \left(\dfrac{100 + \text{Profit}\%}{100}\right) \times \text{C.P.}$
8. S.P. $= \left(\dfrac{100 - \text{Loss}\%}{100}\right) \times \text{C.P.}$
9. Error Based Profit%
$$= \frac{\text{Error}}{\text{Value Given}} \times 100$$
10. Comparisions of S.P. (Find IInd S.P.)

$$\frac{100 \pm P\% / L\%}{S.P._1} = \frac{100 + P\% / L\%}{S.P._2}$$

11. On Purchasing $\rightarrow$ X% Cheat
    On Selling $\rightarrow$ Y% Cheat

    Total P% $= x + y + \dfrac{xy}{100}$
12. C.P. of a = S.P. of b

    it >a7 b then P% $= \left(\dfrac{a - b}{b}\right) \times 100$
13. If b < a, Then L% $= \left(\dfrac{b - a}{b}\right) \times 100$
14. If C.P. = Loss% and S.P. is given then

    C.P. $= \sqrt{25 - S.P.} \times 10 - 50$
15. If C.P. = Profit % and S.P. is given

    then C.P. $= \dfrac{\text{Loss / More Value 10,000}}{(100 + M/L\%)(100 + 2nd\ P) - (100 + 1st\ P)100}$
16. C.P.$_1$ = C.P.$_2$ P = L
    Then no profit and no loss
17. If CP$_1$ = CP$_2$ and by selling then Profit % = Loss%
    Then in whole transaction = No Profit and No Loss.
18. If SP$_1$ = SP$_2$ and P% = L%.
    Then in whole transaction =

    $$\frac{L\%= P^2\%}{100} = \frac{L^2\%}{100}$$
19. C.P. $= \dfrac{\text{Loss / More Value 10,000}}{(100 + M/L\%)(100 + 2nd\ P) - (100 + 1st\ P)100}$

Example 1: A person sells an article for a price which gives him a profit of 20% on cost proce of Rs. 500. Calculate the selling price of the article.

Solution: SP $= CP + \text{Profit}$

$\qquad\qquad = CP + 20\%$ of CP

$\qquad\qquad = CP\left(1 + \dfrac{20}{100}\right)$

$$= \frac{CP \times 120}{100}$$

$$= 1.2 \times CP$$

$$= 500 \times 1.2 = Rs.\ 600$$

$\therefore$ Selling price of the article = Rs. 600

Hence, we conclude that, iff gain percentage is given alongwith the CP then

$$SP = \left[1 + \frac{\% \, gain}{100} \right] \times CP = \frac{(100 + gain \%) \times CP}{100}$$

In case of loss the following formula are applicable

Loss = CP - SP

Loss percentage $\dfrac{Loss}{CP} \times 100$

Like Profit, Loss percentage is also calculated on CP unless specified.

Exaple 2 A person buys an article for Rs. 600 and sells the same at a loss of 30%. Find the selling price of the article.

Solution: SP = CP + Loss

$$= CP + 30\% \text{ of } CP$$

$$= CP \left(1 - \frac{30}{100} \right)$$

$$= \frac{CP \times 70}{100} = 0.7 \times 600 = Rs.420$$

Therefore, the selling price of the article is Rs. 420.

Hence, we conclude that if loss percentage is given alongwith the CP then.

$$SP = \left[1 - \frac{\% \, Loss}{100} \right] \times CP = \left(\frac{100 - Loss\%}{100} \right) \times CP$$

Marked Price: In a sale transaction, the seller marks the goods more than the cost price in order to earn a profit. This addition to the cost price is called the mark up price and this mark up value added to cost price is called the marked price.

Therefore, Marked price = CP+Mark up price.

Now the seller may sell the product on the marked price. In such case,

Marked price = Selling price

He may also sell the product after allowing a discount on the marked price.

In such case Selling price = Marked price-Discount.

Example 3 A person marks his goods 20% more than the cost price and allows some dixcount on it. He still makes a profit of 10%. Find the discount percentage.

Solution:

Let the CP of the article be Rs. 100

Then, MP of the article = Rs. 120

Since Profit = 10% i.e. Rs. 10 (Profit is always on CP hence profit = 10% of Rs. 100=Rs. 10)

$\therefore$ SP=CP+Profit= 100+10 = Rs. 110

The difference between marked price and selling price is the discount.

$\therefore$ Discount=MP-SP=120-110=Rs. 10

Discount %

$$= \frac{Discount}{Marked\ Pr ice} \times 100 = \frac{10}{120} \times 100 = 8.33\%$$

Discount percentage is always calculated on marked price unless specified.

Example 4: A person sells an article for Rs. 360 at a loss of 10%. For what price should he sell the some article to gain 20%.

Solution: Since, he sells the article at a loss off 10%. It means that selling price is 90% of the cost price.

$$\left\{ SP = \left(\frac{100 - loss\%}{100} \right) \times CP \right\}$$

$\therefore$ SP=0.9×CP

$\Rightarrow$ CP $= \dfrac{SP}{0.9} = \dfrac{360}{0.9} =$ Rs.400

Cost price = Rs. 400

Now, if 20% prifit is to be earned then selling price will be 120% CP.

{SP=CP+20% of CP)

$\therefore$ SP=120 of 400=1.2×400

= Rs. 480

Therefore in order to earn a profit 20%, the product should be sold for Rs. 480.

False Weight

Example 5: A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 g for app kg weight. Find his gain percent.

Solution: Suppose the cost price of 1 kg of good is Rs. 100.

Therefore cost price of 960g (using a false weight instead of 1 kg), the selling price of 1kg would be i.e., Rs. 100.

Therefore, profit = SP-CP

= (100-96) = Rs. 4

Hence,

Profit % $= \dfrac{Profit}{CP} \times 100 = \dfrac{4}{96} \times 100 = 4\dfrac{1}{6}\%$

IInd Method

Gain % $= \left[\dfrac{Error}{(trueValue)-(Error)} \times 100 \right]\%$

This farmula can be used in questions of false weight.

$\therefore$ Gain % $= \left(\dfrac{40}{960} \times 100 \right)\% = 4\dfrac{1}{6}\%$

Example 5: A dishonest dealer professes to sell his goods at cost price. But the uses a false weight and thus gains $6\dfrac{18}{47}\%$. Find the weight

he uses in place of 1kg.

Solution: We know,

gain% $= \left[\dfrac{Error}{True\, value - Error} \times 100 \right]$

Let the error be x gram,

then $\dfrac{300}{47} = \left(\dfrac{x}{1000-x} \right) \times 100$

$\Rightarrow$ $\dfrac{3}{47} = \dfrac{x}{1000-x}$

$\Rightarrow$ 3000-3x=47x

$\Rightarrow$ x=60g

$\therefore$ Weight used by the dealer = (1000-60)=940 g

Example 7: By selling two articles for Rs. 180 each a shopkeeper gains 20% on one and looses 20% on the other, find the percentage profit/loss.

Solution: SP=CP+20% (Profit) and SP=CP-20% (loss)

$\therefore$ 180=1.2×CP and 180 =0.8 ×CP

$\therefore$ C.P.=150 and C.P. = 225

This is the cost price of the article for which 20% profit is earned.   this is cost of the article for which 20% loss is occured.

Total cost price =(150+225)=Rs.375

Total selling price= (180+180)=Rs. 360

Hence loss=CP-SP=(375-360)=Rs. 15

and loss % $= \dfrac{15}{375} \times 100 = 4\%$

In cases, where the selling price of two article is some and one is sold of the loss of x% and another is sold at a profit of x%, or in other words, the profit % and loss% is some and selling price is some. This transaction always yields a loss and such loss%.

$= \left(\dfrac{common\, loss\, or\, gain\%}{10} \right)^2$

in the above case loss % $= \left(\dfrac{20}{10}\right)^{2} = 4\%$

Example 10: By selling 20 maggoes, a person recovers the cost price of 25 mangoes. Find the goin or loss percentage.

Solution: Given, SP of 20 mangoes

= CP of 25 mangoes.

Now, suppose SP of 20 mangoes

= CP 25 mangoes = Rs. 100

$\therefore$ SP of 1 mango = Rs. $\dfrac{100}{20}$ =Rs. 5 and

CP of 1 mango.

$$= Rs. \dfrac{100}{25} = Rs. 4$$

Since SP>CP hence this transaction will yield a profit.

$\therefore$ Profit=Rs (5-4)=Re 1 and Profit% =

$\dfrac{1}{4} \times 100 = 25\%$

Example 11: Toffees are bought at 12 for a rupee and are sold 10 for a rupee. Find the percentage profit or loss.

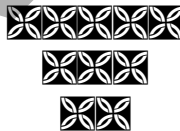**Solution:** CP of 12 toffees = SP of 10 toffees

= Re 1

$\therefore$ CP of 1 toffees= Re $\dfrac{1}{12}$ and SP

of 1 toffee - Re $\dfrac{1}{10}$

Since SP>CP hence there is a gain in this transation.

Gain = SP-CP=Re $\left(\dfrac{1}{10} - \dfrac{1}{12}\right)$ = Re $\dfrac{1}{60}$

Gain% = $\dfrac{\dfrac{1}{60}}{\dfrac{1}{12}} \times 100 = 20\%$

1. A man buys 25 chairs for Rs. 375 and sells them at a profit equal to the selling price of 5 chairs. What is the selling price of one chair?

   (a) Rs. 18.75 (b) Rs. 14.50

   (c) Rs. 15.20 (d) Rs. 17.20

2. The cost price of 9 articles is equal to the selling price of 11 article. Find the loss percentage.

   (a) $18\dfrac{2}{11}\%$ (b) $2\dfrac{9}{11}\%$

   (c) $15\dfrac{1}{2}\%$ (d) $16\dfrac{1}{2}\%$

3. A dishonest seller uses a weight of 800 g in place of 1 kg and adds 20% impurities in sugar. What would be his profit percentage if he chaims to sell the goods at thecost price?

   (a) 50% (b) 40%

   (c) 25% (d) 45%

4. If the cost price of 36 books is equal to the selling price of 30 books, then the gain percentage is.

   (a) 20% (b) $16\dfrac{4}{6}\%$

   (c) 16% (d) $8\dfrac{2}{6}\%$

5. A business man marked the price of his goods 30% more than his cost price. He then sells 1/4 of his stock at a discount of 15% and half of the stock at the marked price, and rest at a discount of 30%. Find his gain percentage.

   (a) 16.5% (b) $15\dfrac{3}{8}\%$

   (c) 14.20% (d) 13.37%

6. A shopkeeper bought some pencils at 2 for Re 1, and an equal number at 3 for Rs. 2. He sold the entire lot at 5 for Rs. 3. Find out his gain or loss percentage.

   (a) $2\dfrac{6}{7}\%\,loss$ (b) $3\dfrac{6}{7}\%\,gain$

   (c) $2\dfrac{6}{7}\%\,gain$ (d) none of these

7. A man sells two articles for the same price for Rs. 640. He earns 20% profit on the first and 10% profit on the second. Find the over all percent profit.

   (a) 14.78% (b) 14.08%

   (c) 14.58% (d) none of these

8. A person incurrs a loss of 10% by selling a toy for Rs. 10.80. At what price he should sell the same to earn a profit of 20%.

   (a) Rs. 12 (b) Rs. 12.96

   (c) Rs. 14.40 (d) None of these

9. A person earn 15% on an investment but loss 10% on another investment. If the ratio of the two investments be 3 : 5, What is the gain or loss on the two investments taken together?

   (a) $6\dfrac{1}{4}\%\,loss$ (b) $13\dfrac{1}{8}\%\,gain$

   (c) $13\dfrac{1}{8}\%\,loss$ (d) None of these

10. A reduction of 25% in the price of eggs enables a man to buy 4 dozen more eggs for Rs. 96. What is the price per dozen?
    (a) Rs. 6   (b) Rs. 8
    (c) Rs. 8.50   (d) Rs. 9

11. A man bought an article and sold it at a gain of 5%. If he had bought it at 5% less and sold it for Re 1 less, he would have made a profit of 10%. The cost price of the article was.
    (a) Rs. 100   (b) Rs. 150
    (c) Rs. 200   (d) Rs. 500

12. A dealer marks his goods 20% above cost price. He then allows some discount on it and makes a profit of 8%. the rate of discount is.
    (a) 4%   (b) 6%
    (c) 10%   (d) 12%

13. Even after allowing a discount of Rs. 32 on an article, a shopkeeper makes a profit of 15%. If the cost price be Rs. 320, What percentage of profit would he heve made if he had sold the transistor at the marked price?
    (a) 20%   (b) 10%
    (c) 25%   (d) None of these

14. A machine is sold at a profit of 10%. Had it been sold for Rs. 40 less, there would have been a loss of 10%. What was the cost price?
    (a) Rs. 175   (b) Rs. 200
    (c) Rs. 225   (d) Rs. 250

15. The retail price of a water geyser is Rs. 1265. If the manufacturer gains 10%, the wholesale dealer gains 15% and the retailer gains 25%, then the cost of the product is.
    (a) Rs. 800   (b) Rs. 900
    (c) Rs. 700   (d) Rs. 600

16. Ravi sells an article at a gain of $12\frac{1}{2}\%$. If he had sold it at Rs. 22.50 more, he would have gained 25%. The cost price of the article is
    (a) Rs. 162   (b) Rs. 140
    (c) Rs. 196   (d) Rs. 180

17. A bought 4 bottles of beer and B bought one bottle of lager, lager per bottle costing twice that of the beer. c bought nothing but paid Rs. 50, for his share of the drink which they mixed together and shared equally. If Cs Rs. 50 covered his share, then what is the cost of the lager?
    (a) 50   (b) 75
    (c) 30   (d) 46

18. Company C sells a line of 25 products with an average retail price of Rs. 1,200. If none of these products sells for less than Rs. 420 and exactly 10 or the priducts sell for less than Rs. 1,000.
    What is the greatest possible selling price of the most expensive products?
    (a) Rs. 2,600   (b) Rs. 7,800
    (c) Rs. 3,900   (d) Rs. 11,800

19. The cost price of two motor cycles is the same. One is sold at a profit of 15% and the other for Rs. 4800 more than the first. If the net profit is 20%. **Fine the cost price of each motor cycle.**
    (a) Rs. 48,000   (b) Rs. 52,000
    (c) Rs. 36,000   (d) Rs. 42,500

20. A delear buys dry fruit at the rate of Rs. 100, Rs. 80 and Rs. 60 per kg. He bought them in the ratio 12 : 20 by weight. He in

total gets 20% profit by selling the first two and at last he finds he has no gain in selling the whole quantity shich had. **What was the percentage loss he suffered for the third quantity?**

(a) 40% (b) 20%

(c) 30% (d) 50%

21. A man sold 250 chairs and had a gain equal to the selling price of 50 chairs. His profit perent is?

(a) 5% (b) 10%

(c) 25% (d) 50%

22. A seller blends two varienties of tea one costing Rs. 18 per kg and another Rs. 20 per kg in the ratio 5 : 3. If he sells the blended variety at Rs. 21 per kg, then his gain percent is?

(a) 10 (b) 12

(c) 22 (d) 19

23. By selling 45 lemouns for Rs. 40, a man loses 20%. How many should he sell for Rs. 24 to gain 20% in the transaction?

(a) 16 (b) 18

(c) 20 (d) 22

24. A bought an article and spent Rs. 100 on its repair. He then sold it to B at 20% profit, B sold it to C at a loss of 10% and C sold it for Rs. 1188 at a profit of 10%. **What is the amount for which A bought the article?**

(a) Rs. 850 (b) Rs. 950

(c) Rs. 930 (d) Rs. 890

25. A horse and a cow were sold for Rs. 12000 each. The horse was sold at a loss of 20% and the cow at a gain of 20%. the entire transaction has resulted in.

(a) loss of Rs. 1000

(b) gain of Rs. 2000

(c) loss of Rs. 500

(d) no loss or gain

26. After allowing a discount of 10% on marked price a shopkeeper charges Rs. 540 for a watch. He he not allowed any discount, he would have made a profit of 20%. What was the cost price of the whach?

(a) Rs. 600 (b) Rs. 500

(c) Rs. 648 (d) Rs. 525

27. A shopkeeper professes to sell all things at a discount of 10%, but increases the selling price of each article by 20%. His gain on each article is

(a) 6% (b) 8%

(c) 10% (d) 12%

28. A shopkeeper earns a profit of 12% on selling a book at 10% discount on the printed price.

The ratio of the ccost price to the printed price of the books is.

(a) 50 : 61 (b) 45 : 56

(c) 99 : 125 (d) 55 : 69

29. A reduction of 20% in the price of sugar enables a coustomer to obtain 2.5 kg more for Rs. 160.

Find the reduced price price per kg.

(a) Rs. 12.8 kg (b) Rs. 15 kg

(c) Rs. 12 kg (d) Rs. 15 kg

30. One mercchant correctly calculates his profit percentage on the cost price, another wrongly calculates it on the selling price.

Find the difference in actual profits if both claim to make 30% prifit, and their revenue is Rs. 3900?

(a) Rs. 270 (b) Rs. 300

(c) Rs. 320 (d) Rs. 350

1. **(a)** SP of 25 chairs = CP of 25 chairs+Profit.

   SP of 25 Chairs = CP of 25 chairs+SP of 5 chairs.

   (∵ profit = SP of 5 chairs)

   ∴   SP of 20 chairs = CP of 25 chairs

   ∴   SP of 20 chairs = Rs. 375

   ∴   SP of 1 chairs = Rs. 18.75

2. **(a)** Let CP of 9 articles = SP of 11 articles = Re 1

   ∴ CP of 1 article = $Re\dfrac{1}{9}$

   SP of 1 article = $Re\dfrac{1}{11}$

   ∴ Loss $= Re\left(\dfrac{1}{9} - \dfrac{1}{11}\right) = Re\dfrac{2}{99}$ and

   $Loss\% = \dfrac{Loss}{CP} \times 100 = \dfrac{\dfrac{2}{99}}{\dfrac{1}{9}} \times 100 - 18\dfrac{2}{11}\%$

3. **(a)** Let the CP of 1 kg of sugar be Rs. 1000.
   After adding 20% impurity in the sugar, the CP of 1200 kg of sugar becomes Rs. 1000. Since he is selling the sugar at the CP then SP of 800 g of sugar will be Rs. 1000. (as he is cheating the buyer to the extent of 20% by way of false weight).

   SP of 800 g of sugar = Rs. 1000

   CP of 800 g of sugar $= Rs\dfrac{1000}{1200} \times 800$

   $= Rs\dfrac{2000}{3}$

   ∴ $Profit\% = \dfrac{Profit}{CP} \times 100$

   $= \dfrac{\dfrac{1000}{3}}{\dfrac{2000}{3}} \times 1000 = 50\%$

4. **(a)** Let the CP of 36 books = SP of 30 books = Re 1

   Then CP of 1 book $= Re\dfrac{1}{36}$ and SP of 1 book $= Re\dfrac{1}{30}$

   $Gain = Re\left(\dfrac{1}{30} - \dfrac{1}{36}\right) = Re\dfrac{6}{30 \times 36}$

   $Gain\% = \dfrac{\dfrac{6}{30 \times 36}}{\dfrac{1}{36}} \times 100$

   $= \dfrac{6}{30} \times 100 = 20\%$

5. **(b)** Let the cost price of 100 items be Rs. 100.

   Marked price of the same= Rs. 130

   SP of 25 items $\left(\dfrac{1}{4} th\right)$ at a discount of Rs. 15%

   $= Rs\dfrac{130}{100} \times 25 \times 0.85 = Rs.27.62$

   SP of 50 items $\dfrac{1}{2}$ at the marked

   Price $= Rs\dfrac{130}{100} \times 50$ =Rs. 65

   SP of 25 items $\left(\dfrac{1}{4}\right)$ at the discount of 30%

$$= Rs. \frac{130}{100} \times 25 \times 0.7 = Rs. 22.75$$

Total selling price = Rs. 115.37

∴ Profit % 15.37 or $15\frac{3}{8}\%$

6. **(c)** Let the shopkeeper bought 5 pencils of each type.

Then, CP of 5 pencils = $Rs. \frac{5}{2}$

and CP of another 5 pencils

$$= Rs \frac{2}{3} \times 5 = Rs \frac{10}{3}$$

∴ CP of 10 pencils = $Rs \frac{35}{6}$

or CP of 5 pencils = Rs. $\frac{35}{12}$

Given, that SP of 5 pencils = Rs. 3

∴ Profit = Rs. $\left(3 - \frac{35}{12}\right) = Rs. \frac{1}{12}$

Profit % $= \frac{1}{12} \times \frac{12}{35} \times \frac{100}{1} = 2\frac{6}{7}\%$

7. **(a)** CP of item sold at a profit of 20%

$$= Rs. \frac{640}{1.2} = Rs. 533.33$$

CP of the item sold at a profit of 10%

$$= = Rs. \frac{640}{1.1} = Rs. 581.81$$

Total CP = Rs 1115.14, total SP = Rs. 1280

$$Profit\% = \frac{164.86}{1115.14} \times 100 = 14.78\%$$

8. **(c)** SP = Rs. 10.80, loss = 10%

then CP = $Rs. \frac{10.80}{0.9} = Rs. 12$

SP at a gain of 20% = Rs 1.2.x12

= Rs.

14.40

9. **(d)** Let the two investments be Rs. 3x and Rs. 5x. Investment after 15% gain.

$$= 3x \times 1.15 = Rs. 3.45x$$

Investment after 10% loss

$$= Rs. 5x \times 0.9 = Rs. 4.50x$$

Total resultant investment taken together = Rs. 7.95x

Loss % $= \frac{0.05x}{8x} \times 100 = \frac{5}{8}\%$

10. **(a)** Reduction of 25% means that due to reduction of Rs. 24 (25% of 96) a man buys 4 dozen more. Therefore reduced price per dozen

$$= Rs. \frac{24}{4} = Rs. 6$$

11. **(c)** Let the CP of the article be Rs. x.

then, SP = 1.05x

New CP = 0.95x

New SP = 1.1 × 0.95x = Rs. 1.045

New it is given that difference between two SP is Rs. 1

∴ 0.005x = Rs. 1 or x = Rs. 200

12. **(c)** Let the CP of the goods be Rx. x.

Then, marked price = Rs. 1.2x

SP of the goods = Rs. 1.08x

Discount = MP - SP = Rs. 0.12x

So, discount % = $= \frac{0.12x \times 100}{1.2x} = 10\%$

13. **(c)** Cost proce of the article = Rs. 320

Selling price of the article

$$= Rs. 1.15 \times 320 = Rs. 368$$

Since, selling price is arived after giving a discount of Rs. 32 on the marked price.

Therefore, marked price = Rs. 400

And if the selling price = marked price

Then profit % $= \frac{400 - 320}{320} \times 100 = 25\%$

14. (b) Let the cost price of machine be Rs. 100

SP of machine at a profit of 10% = Rs. 110

SP of machine at a loss of 10% = Rs. 90

If SP is Rs. 20 less, then CP = Rs. 100

If SP is Rs. 40 less,

then CP $= \dfrac{100}{20} \times 40 = $ Rs.200

15. (a) Let the CP of geyser be Rs. x, then x $\times 1.1 \times 1.15 \times 1.25 = $ Rs.1265

$\Rightarrow \quad x = \dfrac{1265}{1.58125} = $ Rs.800

16. (d) Let the CP of the article be Rs. 100

SP at a gain of $12\dfrac{1}{2}\% = 100 \times 1.125 = $ Rs. 112.50

SP at a gain of 25% = Rs. 125

If SP is Rs. 12.5 more then CP = Rs. 100

If SP is 22.5 more then CP $= \dfrac{100}{12.5}$

$\times 22.5 = $ Rs. 180

17. (a) Total amount spent by them = Rs. 150

Let the cost of one beer be x, then the cost of one lager will be 2x.

total money spent on beer and lager $= (4x+2x) = 6x$

$6x = 150 \Rightarrow x = \dfrac{150}{6} = $ Rs 25

∴ The cost of the lager $= 25 \times 2 = 50$

18. (d) Total selling price of 25 products $= 25 \times 1,200 = $ Rs. 30,000

Price of 24 articles $= 420 \times 10$

$+ 1,000 \times 14$

∴ Price of most expensive product

$= 30,000 - 18,200 = $ Rs. 11,800

19. (a) Let CP of each motor cycle be Rs. x

Then, 2 (1.15x) + 4800 = 2 (1.2x)

or $\quad\quad$ 0.1x = 4800

or $\quad\quad$ x = Rs. 48000

20. (a) Profit earned on first two

$= 20\% (100 \times 12 + 80 \times 15) = 480$

This is the loss on the third type, since he does not make any profit. So loss % =

$\dfrac{480}{(60 \times 20)} \times 100 = 40\%$

21. (c) SP of 250 chairs = CP of 250 Chairs + SP chairs or SP of 200 chairs = CP of 250 chairs.

$\quad\quad\quad\quad\quad\quad = $ Rs. 100 (say)

Then, SP of 1 chair

$= \text{Re}\,\dfrac{1}{2}$ and CP of 1 chair $= \text{Re}\,\dfrac{2}{5}$

$\text{Gain} = \text{Re}\left(\dfrac{1}{2} - \dfrac{2}{5}\right) = \text{Re}\,\dfrac{5-4}{10} = \text{Re}\,\dfrac{1}{10}$

∴ $\text{Gain} = \dfrac{1}{10} \times \dfrac{5}{2} \times 100 = 25\%$

22. (b) Let the quantity of each type of tea be 2x and 3x.

Cost price of 8x kg of tea.

$\quad\quad\quad\quad = $ Rs. $(5x \times 18 + 3x \times 20)$

$\quad\quad\quad\quad = $ Rs. 150x

SP of 8x kg of tea

$\quad\quad\quad\quad = $ Rs. $21 \times 8x = $ Rs. 168x

Profit $\quad = $ Rs. 18x

$\text{Profit}\,\% = \dfrac{\text{Profit}}{\text{CP}} \times 100 = \dfrac{18x}{15x} \times 100$

$\quad\quad\quad = 12\%$

23. (b) SP of 1 lemon = $\text{Re} \dfrac{40}{45} = \text{Re} \dfrac{8}{9}$

loss = 20%

Hence CP = $\dfrac{SP}{0.8} = \dfrac{8}{9 \times 0.8}$

$= \text{Rs.} \dfrac{10}{9}$

SP of 1 lemon in order to gain 20%

$= \text{Rs.} 1.2 \times \dfrac{10}{9} = \text{Rs.} \dfrac{4}{3}$

Therefore in Rs. $\dfrac{4}{3}$ he has to sell 1 lemon to get a profit of 20%

Hence, in Rs. 24 he should sell $\dfrac{3}{4} \times 24 = 18$ lemons.

24. (d) CP of the article for

C = Rs. $\dfrac{1188}{11}$ = Rs 1080

CP of C is the SP of B.

CP for B= = Rs $\dfrac{1080}{0.9}$

Rs. 1200

Similarly CP for A=Rs. $\dfrac{1200}{12}$ = Rs 1000

Since A spends Rs. 110 on repair, therefore original CP = Rs. 890

25. (a) CP of horse $= \text{Rs.} \dfrac{12000}{0.8} = \text{Rs.} 15000$

CP of cow $= \text{Rs.} \dfrac{12000}{1.2} = \text{Rs.} 10,000$

Total CP = Rs. 25,000

Total SP = Rs. 24,000

$\therefore$ loss = Rs. 1,000

26. (b) Marked price of the article

$= \text{Rs.} \dfrac{540}{0.9} = \text{Rs.} 600$

If Marked price = selling price, then there is a Profit of 20%

Therefore, $CP \times 1.2 = \text{Rs.} 600$

or $\quad CP = \text{Rs.} 500$

27. (b) Let the CP of the article be Rs. 100, then SP= Rs. 90

If SP is increased by 20% then new SP=Rs. 108. therefore profit %=8%

28. (b) Let the cost price of the book be Rs. 100

Then, SP at a profit off 12%=Rs.112

$\therefore$ Print Price = $= \text{Rs.} \dfrac{112}{0.9} = \text{Rs.} \dfrac{1120}{9}$

$\therefore \dfrac{\text{Cost Price}}{\text{Print Price}} = \dfrac{100}{\dfrac{1120}{9}}$

$= 900 : 1120 = 45 : 56$

29. (a) Reduction of 20% in the Price means that for reduction amount i.e. for Rs. 32 an additional 2.5 kg may be bought.

$\therefore$ Reduced price per kg= $\text{Rs} \dfrac{32}{2.5}$

= Rs. 12.8
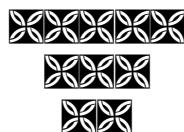
30. (a) Cost price of the article = $\text{Rs.} \dfrac{3900}{1.3}$

Cost price = Rs. 3000 and Selling price = Rs. 3900

Difference in profit = 30% on 3900- 30% on 3000 = Rs. 270

The average of a given set of numbers is a measure of the central tendency puf the set. In other words, it is the mean value of a set of numbers or values. Therefore, average of a set of numbers is given by.

$$Average = \frac{(x_1+x_2+x_3+....+x_n)}{n} = \frac{\sum nx}{\sum n}$$

Where, $\sum nx$ = sum of number and

$\sum n$ = number of numbers

or in other words average of some

$$observations = \frac{Sum\,of\,all\,observations}{Number\,of\,observations}$$

AVERAGE

1. $Average = \dfrac{Total(Sum\,of\,no.)}{Number}$

2. Total = Average x number

3. Average of $1+2+3+$---------$+n=$ $= \dfrac{n+1}{2}$

4. Average of $1^2+2^2+3^2+$-------$+n^2$
   $$= \frac{(n+1)(2n+1)}{6}$$

5. Average of $1^3+2^3+3^3+$-------$+n^3$
   $$= n\left(\frac{n+1}{2}\right)^2$$

6. Average of all consecutive successive
   number $= \left(\dfrac{First+Last}{2}\right)$

7. Average Speed $= \dfrac{Total\ Dis\tan ce}{Time\,Taken}$

8. Average of 2 number $= x$

   First number $= y$

   Seconed number $= 2x-y$

9. Average Speed $A\dfrac{X\,km/h}{Y\,km/h}B$

   $$= \frac{2xy}{x+y}$$

10. Average of A + B + C = 50 kg

    Average of B + C + D = 60 kg

    Total (A+B+C+D)

    = can't be determined

    Since 'B' is repeated itself twice

11. Average of 15 number $= 18$ if 3
    is added to each of them then
    the average will be $= 18+3 = 21$

12. $- 3, - 3, - 3, - 3 = 15$

13. $x3, x3, x3, x3 = 54$

14. $\div3, \div3, \div3, \div3 = 18\div3=6$

15. Average of first 'n' even
    number = n+1

16. Average of first 'n' odd number = n

17. Average of 20 multiples of 5 = $\dfrac{F+L}{2}$

    $$= \frac{5+100}{2}$$

    $$= 525$$

Example 1 if the age of 4 students are 20 yr, 22yr, 18 yr and 24 yr, then what is the average age of the students?

Solution:

By the above definition average age

$$= = \frac{20+22+18+24}{4} = \frac{84}{4} = 21 \text{yr}$$

Therefore, average age of 4 students = 21 yr

WEIGHTED AVERAGE

The concept of weighted average is used when we have two or more groups whose individual averages are known.

Suppose in a class, there are 2 students of 20 yr, 3 of 21 yr, 4 of 22 yr and 5 of 23 yr, then average is given by.

$$\frac{(2 \times 20)+(3 \times 21)+(4 \times 22)+(5 \times 23)}{2+3+4+5}$$

$$= \frac{2}{14} \times 20 + \frac{3}{14} \times 21 + \frac{4}{14} \times 22 + \frac{5}{14} \times 23 = \frac{306}{14} \text{yr}$$

Here $\frac{2}{14}, \frac{3}{14}, \frac{4}{14}$ and $\frac{5}{14}$ are called the

weight of each category of students.

In other words, weights are the fractions of the numbers in that category with respect to the total students in that class. This average is also called the weighted average of that class.

Example 2: What is the average concentration of a mixture if 3L of 36% sulphuric acid is added to 9L of 24% sulphuric acid solution?

Solution: The average concentration of the combined mixture is the weighted average.

$$= \left(\frac{3}{12}\right) \times 36 + \left(\frac{9}{12}\right) \times 24 = 9+18 = 27\%$$

AVERAGE SPEED

$$\text{Average speed - } \frac{\text{Total distance}}{\text{Total Time}}$$

Let the distance between two points A and B is d and speed in travelling from point A to B is x km/h and from point B to A is y km/h.

Then average speed $= \frac{\text{Total distance}}{\text{Total Time}}$

$$= \frac{2d}{\frac{d}{x} + \frac{d}{y}}$$

$$= \frac{2xy}{x+y}$$

If two speeds are given as x km/h and y km/h, then average speed (distance being same)

$$= \frac{2xy}{x+y}$$

Example 3: If a person travels two equal distances at 10 km/h and 30km/h. What is the average speed for the entire journey.

Solution:

$$\text{Average speed} = = \frac{2 \times 30 \times 10}{30+10} = \frac{600}{40} = 15 \text{km/h.}$$

AGE AND AVERAGE

If the average age of n persons decreases by x years. Then the total age of n persons decreases by $(n \times x)$ years. Also, if the average age of n persons increases by x years. Then the total age of n persons increases by $(n \times x)$ years.

Example 4: The average weight of 6 men decreases by 3 kg when one of them weighting 80 kg is replaced by a new man. Calculated the weight of the new man.

Solution: total weight reduced of 6 men= $6 \times 3 = 18$ kg.

This weight of the group is reduced because the man weighting 80 kg is replaced by a man who is 18 kg lighter than him. therefore, weight of new man= (80-18)=62kg.

Example 5: The average of 6 persons is increased by 2 yr. When one of them, whose age is 26 yr is replaced by a new man. What is the age of new person?

Solution: total age increased $= 6 \times 2 = 12$ yr

Age of new persons $= (26+12) = 38$ yr

Because the increase in the total age of 6 persons is due to the replacement of a person aged 26yr with a person who is 12 yr older to him.

Example 6: The average age of 24 boys and the teacher is 15yr. When the teacher's age is excluded, the average decreases by 1. What is the age of teacher?

Solution: Total original age of 25 members $= 15 \times 25 = 375$ yr

Total new age of 24 member $= 14 \times 24 = 336$ yr

$\therefore$ Age of teacher $= (375-336) = 39$ yr

Example 7: The average age of 40 students of a class is 15 yr. When 10 new students are admitted, the average is increased by 0.2 yr. Find average age of the new students.

Solution: Total age of 40 students

$= 40 \times 15 = 600$ yr

New total age of 50 students $= 50 \times 15.2 = 760$ yr

$\therefore$ Total age of 10 students

$= (760-600) = 160$ yr

$\therefore$ Average age of 10 students

$= \dfrac{160}{10} = 16$ yr

RUN AND AVERAGE

Example 8: A cricketer has a certain average of 9 innings. In the tenth inning, he scores 100 runs, thereby increasing his average by 8 runs, calculate his new average.

Solution: Let the average of 9 innings be x rund, hence new average will be (x+8) runs.

Total runs scored for 9 innings $= 9x$

Total runs scored for 10 innings $= 9x+100)$

$$\text{Average for } 10 \text{ innings} = \frac{\text{Total runs}}{10}$$

$$\therefore \quad (x+8) = \frac{(9x+100)}{10}$$

Therefor new average $= (20+8) = 28$ runs

$\therefore \qquad x = 20$

Example 9: A man whose bowling average is 12.4, takes 5 wickets for 26 runs and there by decreases his average by 0.4. Calculate the number of wickets, taken by him before his last match.

Solution: Let the number of wickets taken by him before his last match be x

Then, total runs given by him before last match $= x \times 12.4$

Total runs after last match $= (12.4x+26)$

$$\left[\because \text{Average} = \frac{\text{Runs}}{\text{wickets}} \right]$$

Total wickets taken after last match $= (x+5)$

$\therefore$ Average after last match $= \dfrac{(12.4x+26)}{x+5}$

Given,

$$12 = \frac{(12.4x+26)}{x+5}$$

$12x+60 = 12.4x+26$

x = 85

AVERAGE OF SOME IMPORTANT SERIES OF NUMBERS

❏ The average of odd numbers from 1 to n is $\dfrac{(\text{last odd numbers} + 1)}{2}$,

Where n= last odd number

Exaple 11 What is the average of odd numbers from 1 to 35?

Solution: $\text{Average} = \dfrac{35+1}{2} = 18$

❏ The average of even numbers from 2 to n is $\dfrac{(\text{last even number} + 2)}{2}$,

Where n= last even number

Example 12 What is the average of even numbers from 2 to 50?

❏ The average of square of natural numbers.

$$\text{average} = \dfrac{(n+1)(2n+1)}{6}$$

Solution:

$$\text{Average} = \dfrac{50+2}{2} = 26$$

Example 13 What is the average of square of the natural numbers from 1 to 10?

Solution: $\text{Average} = \dfrac{(10+1)(20+1)}{6} = 38.5$

❏ The average of cubes of natural numbers till n is $= \left[\dfrac{n(n+1)^2}{4}\right]$

Example 14 Find the average of squares of consecutive even numbers till 10.

Solution:

$$\text{Average} = \dfrac{(10+1)(10+2)}{3} = \dfrac{11 \times 12}{3} = 44$$

❏ The average of square of consective odd numbers till n is $\dfrac{n(n+2)}{3}$
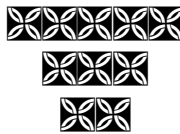
Exercise

1. The average of marks of a student in 7 subjects is 75. His average in 6 subjects excluding science is 72. How many marks did hegetin science?

   (a) 72   (b) 90

   (c) 93   (d) none of these

2. The average age of 30 students of a classis 12yr. The average age of a group of 5 of the students is 10 yrand that of another group of 5 of them is 14yr. What is the average age of theremaining students?

   (a) 8 yr   (b) 10 yr

   (c) 12 yr   (d) 14 yr

3. the average weigh of 9 mangoes increases by 20 g if one of them weighing 120 g is replaced by another. The weight of new mango is.

   (a) 180 g   (b) 200 g

   (c) 260 g   (d) 300 g

4. The average age of a class is15.8 yr. The average age of the boys in the class is 16.4 yr while that of the girls is 15.4 yr. What is the ratio of boys to girls in the class?

   (a) 1 : 2   (b) 3 : 4

   (c) 3 : 5   (d) none of these

5. A motorist travels to a place 150 km away at an average speed of 50 km/h and returns at 30 km/h. His average speed for the whole journey in km per hour is

   (a) 35   (b) 37

   (c) 37.5   (d) 40

6. The average weight of 29 students is 28 kg. By the admission of a new students, the average weight is reduced to 27.8 kg. The weight of the new student is.

   (a) 22 kg   (b) 21.6 kg

   (c) 22.4 kg   (d) 21 kg

7. A team of 8 persons joins in a shooting competition. The best marksman scored 85 points. If he had scored 92 points, the average score for the team would have been 84. The number of points, the team scored was.

   (a) 672   (b) 665

   (c) 645   (d) 588

8. Five years age, the average age of P and Q was 15 yr. Average age of P, Q and R today is 20yr. How old will be after 10yr?

   (a) 35yr   (b) 40yr

   (c) 30yr   (d) 50yr

9. Five years ago, the average age of A, B, C and D was 45yr. With E joining them now, the average of all the five is 49 yr. How old is E?

   (a) 25yr   (b) 40yr

   (c) 45yr   (d) 64yr

10. The average age of a committee of eight members is 40 yr. A member ages 55yr

retired and his place was taken by another member aged 39yr. The average age of the present committee is.

(a) 39yr   (b) 38yr

(c) 36yr   (d) 35yr

11. In a Cricket Eleven, the average age of eleven players is 28yr. Out of these, the average ages of three groups of three players each are 25yr, 28yr and 30yr respectively. If in these groups, the captain and the youngest players are not included, and the captain is eleven years older than the youngest player, what is the age of the captain?

(a) 33yr   (b) 34yr

(c) 35yr   (d) 36yr

12. In seven given numbers, the average of first four numbers is 4 and that of last four numbers is also 4. If the average of these seven numbers is 3, the fourth numbers is

(a) 3   (b) 4

(c) 7   (d) 11

13. Mukesh has twice as much money as Sohan and Shohan has 50% more money than what Pankaj has. If the average money with them is Rs. 110, then Mukesh has.

(a) Rs. 55   (b) Rs. 60

(c) Rs. 90   (d) Rs. 180

14. A person covers half his journey by train at 60km/h, half the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. Find his average speed during the entire journey.

(a) 15 km/h   (b) 24 km/h

(c) 20 km/h   (d) 33.3 km/h

15. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.8 kg. If the average monthly consumption per head be 15kg in the case of males and 6kg in the case of females.

Find the number of females in the family.

(a) 8   (b) 7

(c) 9   (d) 15

16. The average temperature on Monday, Tuesday and Wednesday was 41^0C and on tuesday, Wednesday and Thursday it was 40^0C. If on Thursday it was exactly 39^0C, then on Monday, the temperature was.

(a) 42^0C   (b) 46^0C

(c) 23^0C   (d) 26^0C

17. The average of x, y and z is 45. x is as much more than the average as y is less than the average. Find the value of z.

(a) 45   (b) 25

(c) 35   (d) 15

18. The mean temperature of Monday to Wednesday was 37^0C and of Tuesday to was 34^0C. If the temperature on Thursday was 4/5 that of Monday, the temperature on Thursday.

(a) 38^0C   (b) 36^0C

(c) 40^0C

(d) date inadequate

19. The average weight of a class of 29 students is 40kg. If the weight of the teacher be included, the average rises by 500g. What is the weight of the teacher?

(a) 48   (b) 45

(c) 51   (d) 42

20. The average age of all the students of class is 24yr. The average age of boys of the class is 29 yr and that of the girls is 20yr. If the number of girls in the class is 25, then find the number of boys in the class.

(a) 30   (b) 15

(c) 24   (d) 20

21. The average score of a cricketer in three matches is 22 runs and in two other matches, it is 17 runs. Find the average in all the five matches.

(a) 20   (b) 19.6

(c) 21   (d) 19.5

22. The average weight of a school of 40 teachers is 80 kg. If however the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?

(a) 109kg   (b) 29kg

(c) 39 kg   (d) none of these

23. The average salary of all the workers of a office is Rs. 2000, the average salary of the officers is Rs. 4000 and the salary of rest of staff is Rs. 1250. The total number of workers could be

(a) 110   (b) 300

(c) 410   (d) 530

24. A demographic survey of 100 families in which two parents were present revealed that the average age A, of the oldest child is 20 years less than- the sum of the ages of the two parents. If F represents the age of one parent and M the age of the other parent, then which of the following is equivalent to A?

(a) $\dfrac{F+M-20}{2}$ (b) $\dfrac{F+M}{2}+20$

(c) $\dfrac{F+M}{2}-20$ (d) $F+M-10$

25. The average (arithmetic mean) of x and y is 40. If z=10, then what is the average of x, y and z?

(a) $16\dfrac{2}{3}$ (b) 30

(c) 25   (d) $17\dfrac{1}{2}$

26. The average of 10 numbers is 40.2. Later itis found that two numbers have been wrongly added. The first is 18 greater than the actual number and the second number added is 13 instead of 31. Find the correct average

(a) 40.2   (b) 40.4

(c) 40.6   (d) 40.8

27. Without any stoppage a person travels a certain distance at an average speed of 42km/h and with stoppages he covers the same distance at an average speed of 28 km/h. How many minutes per hour does he stop?

(a) 14min   (b) 15min

(c) 28min   (d) 20min

28. A school has 4 sections of CChemistry in Class X having 40, 35, 45 and 42 students. The mean marks obtained in Chemistry test are 50, 60, 55 and 45 respectively for the 4 sections. Determine the overall average of marks per student.

(a) 50.25   (b) 52.25

(c) 51.25   (d) 53.25

29. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many paper were there in the examination.

(a) 9   (b) 8

(c) 10   (d) 11

30. My Scooty gives an average of 40 km/L. of petrol. But after recent filling at the new petrol pump, its average dropped to 38km/L. I investigated and found out that is was due to adulterated petrol. Petrol pump add Kerosene, which is 2/3 cheaper than petrol. Petrol pump add Kerosene, which is 2/3 cheaper than petrol, to increase their profits. Kerosene generates excessive smoke and knowcking an d gives an average of 18 km per 900 ml. If I paid Rs. 30 for alitre of petrol, what was the additional amount the pump-owner was making?

(a) Rs. 1.75   (b) Rs. 1.80

(c) Rs. 2.30   (d) Rs. 2
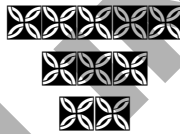
Solution

1. (c) $\text{Average} = \dfrac{\text{Sum of numbers in all subjects}}{\text{Number of Subjects}}$

∴ Sum of numbers in 7 subjects

$= 75 \times 7 = 525$

Sum of numbers in 6 subjects (excluding science) $= 72 \times 6 = 432$

∴ Marks in science $= (525 - 432) = 93$

2. (c) Total age of 30 students

$= 30 \times 12 = 360 \text{yr}$

Total age of 10 students

$= (10 \times 5 + 14 \times 5) = 120 \text{yr}$

Total age of remaining 20 students

$= (360 - 120) = 240 \text{ yr.}$

Average age of 20 students $= \dfrac{240}{20} = 12 \text{yr}$

3. (d) Total increse in weight of 9 mangoes $(20 \times 9) = 180$g. This increase in the weight is due to replacement of a mango with a new whose weight is 180g more than the mango replaced. Therefore, weight of new mango $= (120 + 180) = 300$g.

4. (d) Let the number of boys and girls in the class be x and y respectively. Then the total age of boys $= 16.4x$ and total age of girls $= 15.4y$.

Total age of entine class $= 15.8 (x+y)$

∴ $\quad 15.8 (x+y) = 16.4x + 15.4y$

$0.6x = 0.4y$

⇒ $\quad x : y = 2 : 3$

5. (c) Average speed

$= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{300}{\dfrac{150}{30} + \dfrac{150}{50}}$

$= \dfrac{300}{8} = 37.5 \text{km / h.}$

6. (a) Toal weight of 29 students

$= 29 \times 28 = 812 \text{kg}$

Toal weight of 30 students

$= 30 \times 27.8 = 834 \text{kg}$

∴ Weight of new students

$= (834 - 812) = 22 \text{kg}$

7. (b) Let the number of points scored by 7 persons be x.

New average ⇒ $\quad \dfrac{x + 92}{8} = 84$

∴ $\quad\quad\quad x = 580$

Number of points scored by the team was $= (580 + 85) = 665$

8. (c) Total age of P and Q today

$= (15 \times 2 + 5 \times 2) = 40 \text{yr}$

Total age of P, Q and R today

$= 20 \times 3 = 60 \text{yr}$

Therefore, present age of R

$= 60 \times 40 = 20 \text{yr}$

Age of R 10 yr $= 20 + 10 + 30 \text{yr}$

9. (c) Total present age of A, B, C and D

$= (45 \times 4 + 5 \times 4) = 200 \text{yr}$

Total present age of A, B, C, D and E

$= 49 \times 5 = 245 yr$

$\therefore$ Present age of E= 245-200)=45yr

10. (b) Total age of 8 members

$= 40 \times 8 = 320$ yr

New age of 8 members

$= (320-55+39) = 304$ yr

Average age of 8 members

$= \dfrac{304}{8} = 38 yr$

11. (c) Total age of 11 players=308 yr

Total age of 9 players in a group of $3 = (25 \times 3 + 28 \times 3 + 30 \times 3) = 249 yr$

Total age of remaining 2 plauyers

$= (308-249) = 59 yr$

Let the age of youngest player be x yr, then age of captain $= (x+11)$

$\therefore$ $x + (x+11) = 59$

12. (d) Fourth number

$= \left[(4 \times 4) + (4 \times 4) \right] - (7 \times 3)$

$= 32-21 = 11$

13. (d) Let Pankaj has Rs. x, then Sohan has Rs. 1.5x and Mukesh has $2 \times 1.5x = 3x$

Total money with all the three

$= x+1.5x + 3x = 5.5x$

Average money $\dfrac{5.5x}{3} = 110 (given)$

$\Rightarrow$ $x = 60$

$\therefore$ Money with Mukesh $60 \times 3 = Rs. 180$

14. (b) Let the total distance travelled be x km.

$$Average\, speed = \frac{Total\, distance}{Total\, time}$$

$$= \frac{x}{\dfrac{x}{2 \times 60} + \dfrac{x}{4 \times 30} + \dfrac{x}{4 \times 10}} = \frac{120}{5}$$

$= 24 km/h$

15. (b) Total consumption of males

$= 8 \times 15 = 120$ kg.

Total consumption of females $= x \times 6 = 6x kg$ (x=numbers of females)

Total average $= \dfrac{120 + 6x}{8 + x} = 10.8$

$\Rightarrow$ $120+6x = 86.4+10.8x$

$\Rightarrow$ $4.8x = 33.6$

$\Rightarrow$ $x = 7$

16. (a) Mon+Tues.+Wed.

$= 41 \times 3 = 123^0 C$

Tues.+Wed. + Thu.

$= 40 \times 3 = 120^0 C$

Thu. $= 39^0 C$ $\therefore$ Tues.+Wed.

$= 120-39 = 81^0 C$

From Eqs. (i) and (iii), Mon.

$= 123- 81 = 42^0 C$

17. (a) Sum of x, y and z $= 45 \times 3 = 135$

Given, $x-45 = 45 - y$

$\therefore$ $x+y = 90$

Hence, $z = 135-90 = 45$

18. (a) Mon+Tu.+Wed.

$= 37 \times 3 = 111^0 C$ (i)

Tu.+Wed. + Th.

$= 34 \times 3 = 102^0 C$ (ii)

Gicen, Th. $= \dfrac{4}{5}$ Mon.

Therefore Eq. (ii) becomes

Tu.+Wed.+ $= \dfrac{4}{5}$ Mon=102^{0}C   (iii)

From Eqs. (i) and (ii), M= 45^{0}C

$\therefore$ Then the temperature on

Thursday $= \dfrac{4}{5} \times 45 = 36^0$C

19 (a) Total increase in age of 8 persons = 16yr. Clearly, the total age of two women is 16 yr more than the total age of two men. Hence total age of women

= (35+45)+16=96yr.

Therefore, average age of two

women $= \dfrac{96}{2} = 48$yr

20. (b) Total present age of the family of 6 members = 6 × 22 = 132 yr.

Total age of the family of 6 members 7 yr ago = (132-7×6)=90yr

$\therefore$ Average age of the family at the birth of the youngest member

$= \dfrac{90}{5} = 18$yr

21. (a) Total score of cricketers in three matches = 22×3 = 66

Total score in other two matches = 17×2=34

Average in the five matches

$= \dfrac{66+34}{5} = 20$

22. (c) Total weight of 40 teachers

= 40 × 80=3200 kg

Total weight of 40 teachers and Principal= 79×41=3239 kg

Therefore, weight of the principal

= (3239-3200) = 39kg.

23. (a) Let the number of officers and staff be x and y respectively.

Then 2000 (x+y) = 4000x+1250y

$\Rightarrow$ x : y = 3 : 8

Hence, total number of works should be a multiple of (3+8) = 11. Therefore, option (a) is the correct answer.

24. (c) It is given in the question that

$A = \dfrac{F+M}{2} - 20$

25. (b) Total of x and y = 40×2=80

$\therefore$ Average of x, y and z

$= \dfrac{80+10}{3} = \dfrac{90}{3} = 30$

26. (a) Sum of 10 numbers = 402

Correct sum of 10 numbers

= 402-13+31-18=402

Hence, new average $= \dfrac{402}{10} = 40.2$

27. (d) 42 km takes 60 min

then 28 km will take

$= \dfrac{60}{42} \times 28 = 40$min

28. (b) Required average

$= \dfrac{40 \times 50 + 35 \times 60 + 45 \times 55 + 42 \times 45}{40+35+45+42}$

= 52.25

29. (d) Let the no of papers be x. Then, [63x+22]/ x=65. On solving, we get x=11.

30. (d) Ratio of Prices off petrol : 3:1.

Average with petrol= 40 and with kerosene = 20 km/L. Applying alligation.

40   20

38

18   2
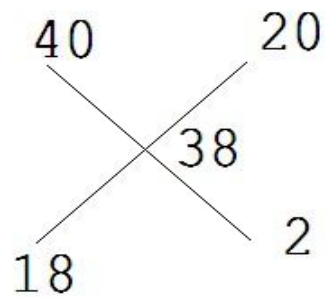
This means that he mixes 1 part kerosene to 9 parts petrol.

In 1L, cost of petrol=30/10 ×9=27 and cost of kerosene = 30 / 10 × 1/3 = Re. 1. Thus adulterated petrol costs Rs. 28 and he makes Rs. 30×28 = 28=Rs. 2 per litre.
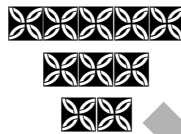
Relationship between speed, distance and time is expressed by speed= $\dfrac{\text{Distance}}{\text{Time}}$ or Distance=Speed×Time. This expression shows that.

1. Speed is directly proportional to distance. If the speed is doubled, then distance travelled in the same time, will also be doubled.

2. Distance and time are directly proportional. If distance to be travelled is doubled, then the time taken would also be doubled at the same speed.

3. Time is inversely proportional to speed. If the distance remains the same and speed is doubled, then time taken to travel the same distance becomes half of the original time taken at the original speed.

Following examples will help to clarify the above relationship.

Distance/ time

1. Speed $= \dfrac{\text{Distance}}{\text{Time}} \text{km / h, M / s}$

     1 km/h $\rightarrow \dfrac{5}{18} \text{m / s}$

     m/s $\rightarrow \dfrac{18}{5} \text{km / h}$

2. Distance = speed x time   km

3. Time = $\dfrac{\text{Distance}}{\text{Speed}}$

4. Average Speed = $\dfrac{\text{Total Distance}}{\text{Total Time taken}}$

5. Average Speed = $A \dfrac{x \text{ km / h}}{y \text{ km / h}} B$

6. $\text{Distance} = \dfrac{S_1 \times S_2}{S_2 - S_1} \times \left(\dfrac{\text{Difference of Timing}}{60} \right)$

Trains

1.
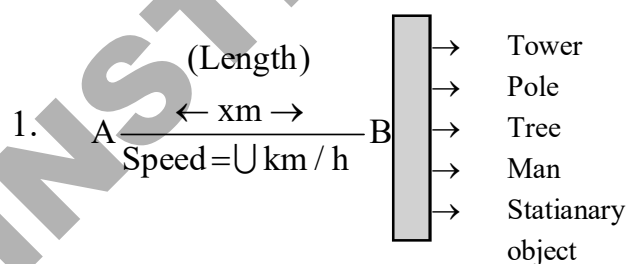
Crossing Time $= \dfrac{x}{\cup \times \dfrac{5}{18}} \text{Sec}$

2. Crossing Time of Bridge / Platform

$$= \left(\dfrac{x + y}{\cup \times \dfrac{5}{18}} \right) \text{Sec}$$

3. $\dfrac{x \text{m}}{\cup \text{km / h}} \rightarrow \leftarrow \left(\dfrac{y \text{m}}{V \text{ km / h}} \right)$

Crossing time (opposite Direction) =

$$\left(\dfrac{x + y}{(\cup + V) \times \dfrac{5}{18}} \right)$$

4. (Crossing time same direction)

$$= \left(\dfrac{x + y}{(\cup - V) \times \dfrac{5}{18}} \right)$$

Example 1 Walking $\frac{5}{6}$ of its usual speed, a train is 10 min too late. Find its usual time to cover the journey.

Solution: Let the usual speed and time be S and T respectively.

Then, New speed $=\frac{5}{6}S$

$\therefore$ New time $=\frac{6}{5}T\left(\therefore \text{Speed a} \frac{1}{\text{Time}}\right)$

Given, (New time) - (Usual time)=10

or $\frac{6}{5}T-T=10\rightarrow\frac{T}{5}=10$ or $T=50$ min

Example 2

If a man at 3m/s, how many kilometres does he run in 1 h 40 min?

Solution: 1 m/s $=\frac{18}{5}$ km/h

$\therefore$ 3 m/s $=3=\frac{18}{5}$ km/h $=\frac{54}{5}$ km/h

$\therefore$ Distance covered in (1 h 40 min $\frac{5}{3}$ h)

$=\frac{54}{5}\times\frac{5}{3}=18$ km

There are certaub stabdard conversion factars

$1 \text{km}/\text{h}=\frac{5}{18}\text{m}/\text{s}$

$1\text{m}/\text{s}=\frac{18}{5}\text{km}/\text{h}$

Example 3 Salabh sees his friend standing at a distance of 200 m from his position. He increase his speed by 50% and hence takes 20s now to reach him.

(a) If he travels at the original speed, how much time will he take?

(b) What was his original speed?

Solution:

(a) Salabh increases his speed by 50% ie, if he originally travelled at speed S and takes time T to cover a distance. Now he travels at 150% ie, at $\frac{3}{2}S$. Therefore, he will now take $\frac{2}{3}T$ time to travel the same distance.

And it is given that $\frac{2}{3}T=20$s

$\therefore$ T=30s

So his original time is 30s.

(b) Original speed

$=\frac{\text{Distance}}{\text{Original time}}=\frac{200}{30}\text{m}/\text{s}$

$=\frac{20}{3}\times\frac{18}{5}=24\text{km}/\text{h}$

Example 4 *If a man walks at the rate of 5km/ h, he misses a train by only 7 min. However, if he walks at the rate of 6km/h, he reaches the station 5 min before the arrival of the train. Find the distance covered by him to reach the station.*

Solution: Difference in the times taken at two sppeds= (7+5)=12 min=$\frac{1}{5}$h

Now let therequired distance be x km.

$$\therefore \frac{x}{5} - \frac{x}{6} = \frac{1}{5} \text{ or } 6x - 5x = 6 \rightarrow x = 6\text{km}$$

$$\left(\because \text{Time} = \frac{\text{Distance}}{\text{Speed}} \right)$$

Example 5 Mr. X arrives at his office 30 min late everyday. On a particular day, he reduces his speed by 25% and hence arrived 50 min late instead. Find how much time would he take to travel his office if hedecides to be on time on a particular day.

Solution: New speed $= \frac{3}{4}S$

$$\therefore \quad \text{New time} = \frac{4}{3}T$$

$$= \frac{4}{3}T - T = 50 - 30$$

or $\quad \frac{T}{3} = 20$

$\Rightarrow$ T=60 min. If was the time taken by X when he was 30 min late. So if he decides to come on time, he would take 30 min to travel.

AVERAGE SPEED

Example 6 *A person goes to Delhi from Meerut at the speed of 60km/h and comes back at the speed of 50 km/h. Calculate the average speed of the person for the entire trip.*

Solution: It is given that distance travelled by the person at different speed in same but time taken by them in travelling both the distances is not known. Therefore, the concept of weighted average will not apply in this case.

Hence, average speed calculated by

$\frac{60 + 50}{2} = 55\text{km} / \text{h}$ does not give the correct average speed.

However this will be correct if it given that time taken by the person in travelling two distances (which will definitely not be equal) is same. Then the above average speed calculation would be correct.

Thus average speed for equal distance is given by.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Now, let the speed of a person in travelling a distance D km to and fro is x km/h and y km/h, respectively.

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2D}{\dfrac{D}{x} + \dfrac{D}{y}}$$

$$= \frac{2xy}{x + y} \text{km} / \text{h}.$$

In travelling equal distance with speeds X and Y, the average speed is expressed as

$$= \left(\frac{2xy}{x + y} \right)$$

In the previous example the average

speed $= = \frac{2 \times 60 \times 50}{60 + 50} = \frac{6000}{110} = 54.54\text{km} / \text{h}.$

RELATIVE SPEED

Relative speed is the speed of a moving object in relation to other moving object.It is

often seen, while sitting in a moving train, speed of the train increases when another train moving in the opposite direction crosses it. The speed appears decreasing when another train moving in the parallel track in the same direction passes it. Hence, this feeling of change in speed of train is nothing but its relative speed in relation to the another moving train.
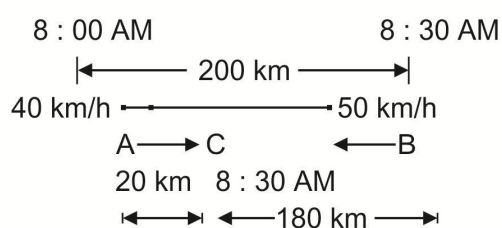
Therefore, relative speed of a moving body with speed x km/h. in relation to the other moving body with speed y km/h.

(a) in same direction is difference of their speeds ie, (x-y) km/h.

(b) in opposite direction is the sum of their speeds ie, (x+y) km/h.

Example 7 *A person starts from city A towards B at 8.00 am at a speed of 40km/h. Another person starts from city B towards A at 8.30 am at a speed of 50km/h. At what time will they meet if the distance between A and B is 200 km?*

Solution: It is clear from the above question that both persons are in motion at 8.30 am.

Hence concept of relative speed will apply only when both the bodies are in motion ie, at 8.30 am. In half hour, person starting from A would have travelled 20km. Hence at 8.30am the distance between A and B is (200-20) =180 km.

```
        8 : 00 AM              8 : 30 AM
        |←——— 200 km ———→|
   40 km/h •————————————• 50 km/h
          A—→ C        ←—B
        20 km   8 : 30 AM
        |←——→| ←——180 km ——→|
```

Distance = 180km, relative speed

= (50+40) = 90 km/h

Hence, time when they meet $= \dfrac{180}{90} = 2h.$

Therefore, 2 h after 8.30 am ie, 10.30 am both the persons will meet.

From the above example it is clear that

1. Concept of relative speed into operations when both the bodies are in motion.

2. Distance between two points has to be calculated at a time when both the bodies come into motion for the first time. In other words, we have to make the time equal if times of start of two bodies are given.

Example 8: *Mohit and Ajay are two friends whose homes are 20km. apart. Both of them decide to meet somewhere between their house. Mohit rides at 8 km/h and Ajay at 10km/h. Mohit leaves his house at 8.00 am and Ajay leaves his house at 9.00 am.*

(a) At what time they meet?

(b) At what distance from Ajay's house do they meet?

Solution: Ajay leaves one hour after Mohit had started.

Hence at 9.00 am. Mohit would have travelled 8 km at a speed of 8 km/h. Now at 9.00 am the distance between Mohit and Ajay= (20-8)=12 km. Which is to be covered at a relative speed of (8+10)=10 km/h.

$$\therefore \text{ Time taken } = \dfrac{12}{18} \times 60 = 40 \min$$

Thus both of them will meet 40 min after 9.00 am ie, at 9.40 am.

(b) Distance travelled by Ajay in 40 min

$$= 10 \times \frac{40}{60} = \frac{400}{60} = \frac{20}{3} \, \text{km or } 6\frac{2}{3} \, \text{km}$$

CONCEPT OF RELATIVE SPEED IN MOTION OF TRAINS

Application of relative speed in solving questions of trains is same as used in the previous examples. Here are given some important points in problems on trains.

1. If two trains with speed km/h and v km/h move in the same directions, then their relative speed= (u-v) km/h, where u>v.

2. If two trains with speed u km/h and v km/h move in the opposite directions, then their relative speed =(u+v) km/h

3. If two trains of length x km and y km are moving in opposite directions at u km/h and v km/h. then time taken by the trains to cross each other $= \left(\dfrac{x+y}{u+v}\right)$ h.

4. If two trains of length x km and y km are moving in the same direction at u km/h and v km/hr, where u>v, then time taken by faster train to cross the slower train $= \left(\dfrac{x+y}{u-v}\right)$ h.

5. Time taken by a train x metres long in passing a single post or a pole or a standing man=Time taken by the train to cover x metres.

6. Time taken by a train x metres long in passing a stationary object of length y metres=Time taken by the train to cover (x+y) metres.

7. If two trains start at the same time from two points A and B towards each other and after crossing they take a and b hours in reaching B and A respectively

Then, A's speed: B's speed $= \left(\sqrt{b} : \sqrt{a}\right)$

Example 9: *A traub 100 m long takes 6s to cross a man walking at 5km/h in opposite direction. Find the speed of train.*

Solution: Let the speed of train be x km/h.

Speed of train in relation to man
= (x+5) km/h

$$= \left(x+5\right) \times \frac{5}{18} \, \text{m/s}$$

Time taken by train to cross the man

$$= \frac{100 \times 18}{\left(x+5\right) \times 5}$$

or $\quad 6 = \dfrac{100 \times 18}{5\left(x+5\right)} \Rightarrow x = 55 \text{m/s}$

Example 10: *A train 140 m long is running at 60 km/h. In how much time will it pass a platform 260 m long?*

Solution: Speed of train $60 \times \dfrac{5}{18} = \dfrac{50}{3} \, \text{m/s}$

Distance covered by train in crossing the platform $=(140+260)=400\text{m}$

$\therefore$ Time taken $= 400 \times \dfrac{3}{50} = 24\text{s}$

BOATS AND STREAM

The problems of boats and streams are also based on the basic relation of speed, distance and time ie.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

In these questions, the direction along the stream (water) is called downstream and, the direction against the stream is called upstream. If the speed of a boat in still water is x km/h and the speed of the stream is y km/h, then

Downstream speed=(x+y) km/h

Upstream speed = (x-y) km/h

Adding (i) and (ii), we get

Downstream speed+Upstream speed=2x

or, $\dfrac{\text{Downstream speed + Upstream speed}}{2}$

$= x = \left(\text{Speed of boat in still water}\right)$

Subtracting (i) and (ii), we get

$\dfrac{\text{Downstream speed-Upstream speed}}{2}$

$= y= $ (Speed or the stream)

From the above relationship we conclude, if the downstream speed is u km/h and upstream speed is v km/h, then

Speed of boat in still water $=\left(\dfrac{u+v}{2}\right)$km/h.

Speed of stream $=\left(\dfrac{u-v}{2}\right)$km / h.

Example 11: *A man rows downstream 24 km, taking 3 h each time. What is the velocity of the current?*
Solution:

Dowbstrean speed $= \dfrac{24}{3} = 8\,\text{km / h}.$

and Dowbstrean speed $\dfrac{18}{3} = 6\,\text{km / h}$

$\therefore$ Speed of stream$=\left(\dfrac{u-v}{2}\right) = \dfrac{8-6}{2}$

$= 1\,\text{km/h}$

CURCYKAR MOTION

The relative motion of two bodies moving around a circle in the same direction is given by (x-y) km/h. Where x and y are the speed of two bodies in km/h.

And, when two bodies are moving ground a circle in the opposite direction, the relative speed of the two bodies is given by (x+y) km/h.

Example 12 *Sumit and Rajan run on a circular track of circumference 600m with a speed of 5km/h and 2 km/h. They start from the same place at the same time and in the same direction. When will they be together again for first time?*

Solution:

Relative speed $=(5-2) = 3 \times \dfrac{5}{18} = \dfrac{5}{6}\,\text{m / s}$

It is clear that Sumit with greater speed will take a lead by a speed which is the relative speed. And with this speed he has to cover a distance of 600 m in order to meet him again for the first time.

$\therefore$ Time taken by Sumit in meating

Rajan $= \dfrac{\text{Distance}}{\text{Relative speed}}$

$= \dfrac{600 \times 6}{5} = 720\text{s}$

Hence, they will be together again after 720s.

FIRST MEETING OF THREE BODIES ON CIRCULAR PATH

In the above example both the bodies will meet after a time which is calculated by dividing the circumference of the circle by the relative speed.

In case when three or more bodies start moving simultaneously from the same point on the circumference of the circle, in the same directions around the circle, they will first meet after a time given by the LCM of the times that the fastest runner takes in totally overlapping each of the slower runners.

FIRST MEETING AT STARTING POINT

In case when three or more bodies start moving simultaneously from the same point on the circumference of the circle, in the same direction around the circle.

All the three bodies will meet again at the starting point after a time which is calculated by taking LCM of the times that each of the bodies takes to complete one full round.

This method is also applicable in case of circular motion of two bodies.

Example 13

Three friends Mukul, Dinesh and Rohan run along a circular park with a speed of 8km/h, 5km/h and 3 km/h respectively. When will they all be together again? The circuference of the park is 500m.

(a) for the first time

(b) for the first time at the starting point.

Solution:

(a) Relative speed of Mukul with respect to Dinesh= (8-5)=3km/h

There, at a relative speed of 3km/h, he would meet Dinesh after every

$$\frac{500}{3 \times \frac{5}{18}} = 600s \quad \text{or 10 min.}$$

Likewise relative speed of Mukul with respect to Rohan = (8-3) = km/h.

Hence at a relative speed of 5km/h, he would meet Rahul after every

$$\frac{500}{5 \times \frac{5}{18}} = 360s \quad =6\text{min.}$$

Therefore, all of three would meet after every LCM of (10, 6)=30 min. Hence, they would all meet for the first time after 30 min.

(b) Time taken by Mukul in making one revolution

$$= \frac{500}{8 \times \frac{5}{18}} = 225s$$

Time taken by Dinesh in making one revolution $$= \frac{500}{5 \times \frac{5}{18}} = 360s$$

and Time taken by Rohan in making one revolution.

$$= \frac{500}{3 \times \frac{5}{18}} = 600s$$

Hence, they would meet for the first time at the starting point after LCM of (225, 360, 600) = 1800s=30 min.
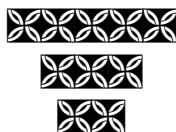
1. Walking at $\frac{3}{4}$ of his normal speed, Abhishek is 16 min late in reaching his office. The usual time taken by him to cover the distance between his home and his office is.

   (a) 48 min   (b) 60 min

   (c) 42 min   (d) 62 min

2. Two trains for Bhopal leave Delhi at 9 am and 8.30 am and travel at 90 km/h and 80 km/h respectively. How many kilometres from Delhi will the two train be together.

   (a) 360km   (b) 320 km

   (c) 270km   (d) 280 km

3. Prabhat covers a distance in 40 minif he drives at a speed of 60 km/h on an average. Find the speed at which he must drive at to reduce the time of the journey by 25%?

   (a) 60 km/h   (b) 70 km/h

   (c) 75 km/h   (d) 80 km/h

4. Without stoppage, a train travels a certain distance with an average speed of 60 km/h, and with stoppage, it covers the same distance with an average speed of 40 km/h. On an average, how many minutes per hour does the train stop during the journey.

   (a) 20 min/h   (b) 15 min/h

   (c) 10 min/h   (d) 12 min/h

5. Ramesh, during his journey, travels for 20 min at a speed of 30 km/h, another 30 min at a speed of 50 km/h, next 1 h at a speed of 50 km/h and 1 h at a speed of 60 km/h. What is the average speed of the entire journey?

   (a) 51.18 km/h   (b) 48 km/h

   (c) 63 km/h   (d) 39 km/h

6. A man riding on a bicycle at a speed of 15 km/h crosses a bridge in 5 min. Find the length of the bridge.

   (a) 1 km   (b) 2 km

   (c) $2\frac{1}{2}$ km   (d) $1\frac{1}{4}$ km

7. A train goes from a station A to another station B at a speed of 64 km/h but returns to A at a slower speed. If its a average speed for the trip is 56km/h, the return speed of the train nearly.

   (a) 48 km/h   (b) 50 km/h

   (c) 52 km/h   (d) 47.4 km/h

8. A man is walking at a speed of 9 km/h. After every km he takes rest for 9 min. How much time will he take to cover a distance of 27 km?

   (a) 6 h   (b) 6 h 45 min

   (c) 6 h 54 min   (d) 6 h 35 min

9. Ram travels at the rate of 3 km/h and he reaches 15 min late. If he travels at the rate of 4 km/h, he reaches 15 min earlier. The distance Ram has to travel is

   (a) 1 km   (b) 6 km

   (c) 7 km   (d) 12 km

10. A man walks to a town at the rate of $5\frac{1}{2}$ km / h and rides back at the rate of 10 km/h. How far he walked, if the total time of the journey is 6 h 12 min?

    (a) 31 km   (b) 29 km

    (c) 22 km   (d) 17 km

11. A motor car does a journey in 9h, the first half at 12 km/h and second half at 15 km/h. Find the distance.

    (a) 120 km   (b) 100 km

    (c) 124 km   (d) 96 km

12. The distance between two stations A and B is 220 km. A train leaves A towards B at an average speed of 80 km/h. After half an hour another train leaves B towards A at an average speed of 100 km/h. Find the distance from A to the point where the two trains meet.

    (a) 180 km   (b) 120 km

    (c) 160 km   (d) 80 km

13. Two men A and B start from a place P walking 4 km and 5km/h respectively. How many km will they be apart at the end of 4h, if they walk in same direction?

    (a) 3 km   (b) 4 km

    (c) 2 km   (d) 4.5 km

14. Two men A and B walk from P to Q, a distance of 22 km, at 5 and 6 km an hour respectively. B reaches Q, returns immediately and meets A at R. Find the distance from P to R.

    (a) 16 km   (b) 18 km

    (c) 20 km   (d) 15 km

15. Without stoppage a person travels a certain distance at an average speed of 15 km/h, and with stoppages he covers the same distance at an average apeed of 12 km/h. How many minutes per hour does he stop?

    (a) 15 min   (b) 12 min

    (c) 16 min   (d) 18 min

16. Sudhakar has to cover a distance of 6 km in 45 min. If he cover one half of the distance in $\frac{2}{3}$ rd time, what would be his speed in km/h to cover the remaing distance in the remaining time

    (a) 12   (b) 16

    (c) 3   (d) 8

17. A man in car completes a journey without stopping in 9 h. If he has travelled 8 km an hour faster, it would have done the journey in 6 h. What is his original speed?

    (a) 27km/h   (b) 16 km/h

    (c) 24km/h   (d) 18 km/h

18. A train 160 m long is running at 40 km/h. In how much time will it pass a platform 140 m long?

    (a) 30s   (b) 24s

    (c) 15s   (d) 27s

19. A train 120 m long is running at a rate of 54 km/h. Time taken by the train to cross a tunnel 130 m long is

    (a) $8\frac{1}{3}$s   (b) $16\frac{2}{3}$s

    (c) 10s   (d) 15s

20. A train 100 m long takes 9s cross a man walking at 5 km/h in the direction opposite to that of the train. Find the speed of the train.

    (a) 40 km/h   (b) 30 km/h

    (c) 45 km/h   (d) 35 km/h

21. Two trains are running on parallel lines in the same directions at speed of 40 km/h and 20 km/h respectively. The faster train crosses a man in the second train in 36s. The length of the faster train is

    (a) 20 m   (b) 185 m

    (c) 225 m   (d) 210 m

22. Two train pass each other on parallel lines. Each train is 100 m long. When they are going in the same direction, the faster one takes 60s to pass the other completely. If they are going in the oppisite directions they pass each other completely in 10s. Find the speed of the slower train in km/h.

    (a) 30 km/h   (b) 42 km/h

    (c) 48 km/h   (d) 60 km/h

23. A good train and a passenger train are running on parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in 60s. Whereas a passenger on the passenger train marks that he crosses the goods train in 40s. If the speeds of the trains be in the ratio 1 : 2, find the ratio of their lengths.

    (a) 2 : 1   (b) 3 : 1

    (c) 1 : 4   (d) 2 : 3

24. A motorboat went down the river for 14 km and then up the river for 9 km. It took a total of 5 hours for the entire journey.

    Find the speed of the boat in still water is 5 km/h.

    (a) 1 km/h.   (b) 1.5 km/h.

    (c) 6 km/h.   (d) 8 km/h.

25. In a stream that is running at 2 km/h, a man goes 10 km upstream and comes back to the starting point in 55 min.

    Find the speed of the man in still water.

    (a) 20 km/h.   (b) 22 km/h.

    (c) 24 km/h.   (d) 28 km/h.

26. The diameter of a cycle wheel is 70 cm. a ccyclist takes 30h to reach a destination at the speed of 22 km/h. How many revolutions will the wheel make during this journey?

    (a) 3 million   (b) 3 Lakh

    (c) 4 Lakh   (d) none of these

27. In a flight of 3000 km an aircraft was slowed down by bed weather. If average speed for the trip was reduced by 100km/h and the time increased by one hour, find the original duration of the flight.

    (a) 5 h   (b) 6 h

    (c) 4 h   (d) none of these

28. A car driver travels from the plains to the hiss station, which are 200 km a part at an average speed of 40 km/h. In the return trip he covers the same distance at an average speed of 20 km/h the car. The average speed of the car over the entire distance of 400 km is.

(a) 16.56km/h.   (b) 17.89 km/h.

(c) 26.67km/h.   (d) 35 km/h

29. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 min?

(a) 648   (b) 1,800

(c) 2,700   (d) 10,800

30. A min sitting in train travelling at the rate of 50 km/h observes that it takes 9s for a goods train travelling in the opposite direction to pass him. If the goods train is 187.5 m long, find its speed?

(a) 40 km/h.   (b) 25 km/h.

(c) 35 km/h.   (d) 36 km/h.
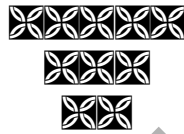
1. **(a)** If the speed $\frac{3}{4}$ th of the normal speed, then time will become $\frac{4}{3}$ of the usual time. Let the usual time be T, then

    $\frac{4}{3}$ T-T=16 $\therefore$ T=48 min

2. **(a)** Distance travelled by the train at 80 km/h in 30 min=40 km. This distance of 40 km is to be covered with a relative speed of (90-80) = 10 km/h.

    $\therefore$ Time taken to cover this distance $\frac{40}{10} = 4h$.

    Hence, the trains will together after 4×90 = 360km from Delhi.

3. **(d)** Distance required to be travelled

    $= 60 \times \frac{40}{60} = 40 km$

    Time available to cover this distance $= 40 \times 0.75 = 30$ min

    $\therefore$ Speed $= \frac{40}{\frac{30}{60}} = 80 km/h$

4. **(a)** Due to stoppages the train travels 20 km less in one hour. and the time taken to travel 20 km is the time of the train taken at stoppages. Therefore time taken to cover 20 km.

    $= \frac{20}{60} \times 60 = 20 \, min/h$

5. **(a)** Average speed $= \frac{Totoal \, distance}{Total \, time}$

    $= \dfrac{30 \times \frac{1}{3} + 50 \times \frac{1}{2} + 50 \times 1 + 60 \times 1}{\frac{1}{3} + \frac{1}{2} + 1 + 1}$

    $= \dfrac{6(10 + 25 + 50 + 60)}{17}$

    $= 51.18$ (App.) km/h

6. **(d)** Lenght of bridge = Speed×time taken to cross the bridge

    $= \frac{15 \times 5}{60} = 1\frac{1}{4} km$

7. **(b)** Average speed $= \frac{2xy}{x + y}$

    and $56 = \dfrac{2 \times 64 \times x}{64 + x}$

    $\Rightarrow 3584+56x = 128x$
    or 50 km/h (nearly)

8. **(c)** Since, he is taking a rest after every one km. therefore time consumed in taking rest in 27 km=26×9=234 min = 39h.

    Time taken to cover 27 km without rest= 3h.

    Total time taken = 3+3.9 =6.9h or 6 h 54 min

9. (b) Time difference = (15+15)=30 min

$$\therefore \quad \frac{x}{60} - \frac{x}{80} = \frac{30}{60}$$

or

$$\frac{x}{12} = \frac{1}{2} \qquad \therefore x = 6km$$

10. (c) Let the distance between two point be x km

$$\frac{2x}{11} + \frac{x}{10} = 6 + \frac{12}{60} = \frac{31}{5}$$

or $\qquad \dfrac{31x}{110} = \dfrac{31}{5} \Rightarrow x = 22km.$

11. (a) Let the half distance be x km.

Therefore, $\dfrac{x}{12} + \dfrac{x}{15} = 9$

or

$$\frac{5x + 4x}{60} = 9 \Rightarrow x = 60km.$$

$\therefore \qquad$ Total distance = 60×2=120km

12. (b) Distance travelled by the train moving from A in $\dfrac{1}{2}h = 40km$

Now the distance of (220-40)

=180 km will be covered by a relative speed of (80+100)=180 km/h

Hence time taken in meeting between

points A and B= $\dfrac{180}{180}$ =1h

Now, distance travelled by the first train in 1h=80 km.

Total distance moved by this train before both the train meet

= (40+80) = 120 km.

13. (b) Distance between two men after 4h= (5×4-4×4) = 4km

14. (c) When B meets A at R, B has walked the distance PQ+QR and A the distance PR. That is both of them have together walked twice the distance from P to Q ie, 44km.

Now the speed of A and B are 5:6

Hence distance PR travelled by A

$$= \frac{5}{11} \times 44 = 20km$$

15. (b) Time at stoppage is equal to the time in travelling (15-12) =3km less in one hour.

And time taken to travel 3 km

$$= \frac{60}{15} \times 3 = 12 \, min$$

Therefore, train stops 12 min per hour.

16. (a) Remaining distance = 3km and Remaining time = 15 min

$\therefore \qquad$ Speed to cover remaining distance in remaining

time $= \dfrac{3}{15} \times 60 = 12km / h$

17. (b) Let the original speed be x km/h, Time = 9 H

Then distance to be covered=9x

New speed = (x+8) km/h.

New time = 6h

Then, $(x + 8) = \dfrac{9x}{6}$

$6x + 48 = 9x$

$x = 16$ km/h

18. **(d)** Total distance $= (160 + 140) = 300$ m,

Speed $= 40 \times \dfrac{5}{18} = \dfrac{200}{18}$ m/s

$\therefore$ Time taken to cross the platform

$= \dfrac{300}{200} \times 18 = 27$ s

19. **(b)** Required time $= \dfrac{250 \times 18}{54 \times 5}$

$= \dfrac{4500}{270}$

$= 16\dfrac{2}{3}$ s

20. **(d)** Let the speed of the train be x km/h

Then relative speed $= (x + 5)$ km/h

Distance $= \dfrac{100}{1000}$ km, Time $= \dfrac{9}{60 \times 60}$ h

Then, Distance = Speed × Time

$\dfrac{1}{10} = (x + 5) \times \dfrac{9}{60 \times 60} \Rightarrow x = 35$ km/h

21. **(a)** Let the length of the faster train be x metres.

Relative speed $= (40 - 20) = 20$ km/h

$x = 20 \times \dfrac{5}{18} \times 36 = 200$ m

22. **(a)** Let the speed of faster train be x km/h and that of slower train be y km/h.

Then, in first case, relative speed $= (x - y)$ km/h.

$\therefore \qquad (100 + 100) = (x -$

$y) \times \dfrac{5}{18} \times 60$

or $\qquad x - y = 12 \qquad$ (i)

In second case, relative speed

$= (x + y)$ km/h

$200 = (x + y) \times \dfrac{5}{18} \times 10$

or $x + y = 72$

(ii)

From Eqs. (i) and (ii), $x = 42$ km/h and $y = 30$ km/h

23. **(a)** Let the speeds of the two trains be x and $2x$ m/s respectively. Also, suppose that lengths of the two trains are A metres and B metres respectively.

Then, $\qquad \dfrac{A + B}{2x - x} = 60 \qquad$ ...(i)

and $\qquad \dfrac{A}{2x - x} = 40 \qquad$ ...(ii)

Dividing Eq. (i) by Eq. (ii) we get

$\dfrac{A + B}{A} = \dfrac{60}{40}$

or $\qquad \dfrac{B}{A} + 1 = \dfrac{3}{2} \qquad$ o r

A:B = 2:1

24. **(c)** Let the sped of the stream be x km/h.

Then downward speed $= (5 + x)$ km/h and downward speed $= (5 - x)$ km/h.

Then, $\dfrac{14}{(5 + x)} + \dfrac{9}{(5 - x)} = 5 \Rightarrow x = 2$ km/h

25. (b) Let the speed of the man in still water be x km/h then,

$$\frac{10}{(x-2)} + \frac{10}{(x+2)} = \frac{55}{60} \Rightarrow x = 22\,\text{km}/\text{h}$$

26. (b) Circumference of the wheel

$$= 2\eta r = 2 \times \frac{22}{7} \times 35 = 220\,\text{cm}$$

∴ Number of revolution made by the wheel during the journey

$$= \frac{2200000 \times 30}{220}$$

(∵ 22km=2200000 cm)

27. (a) Let the original time taken by the plane be x hours.

Then, original speed $= \dfrac{3000}{x}\,\text{km}/\text{h}$

∴ $\dfrac{3000}{x+1} = \dfrac{3000}{x} - 100$

$\Rightarrow$ x=5

28. (c) Average speed $= \dfrac{2xy}{x+y}$ where x and y are two different speeds.

∴ Average speed

$$\frac{2 \times 40 \times 20}{(20+40)} = 26.67\,\text{km}/\text{h}$$

29. (b) $m_1 \times t_1 \times w_2 = m_2 \times t_2 \times w_1$

$6 \times 1 \times x = 10 \times 4 \times 270$

$\Rightarrow$

$$x = \frac{10 \times 4 \times 270}{6} = 1800$$

30. (b) Let the speed of the goods train be x km/h, than

$$(50 + x)\frac{5}{18} = \frac{187.5}{9}$$

$\Rightarrow$ x=25km/h
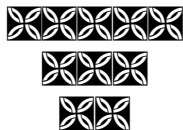
6 Simple Interest and Compound Interes

Simple Interes

Simple mterest is the interest accrued on a certain sum at a certain rate of interest on flat basis irrespective of any time. It means for first and subsequent years the amount on which the interest is calculated remains the same and hence no benefit on the interest calculated on the previous years is given in the subequent years.

Simple interest is given by the following formula

$$SI = \frac{P \times R \times T}{100}$$

Where P = Principal amount on which the interest is calculated

R = rate of interest (per annum)

T = time period for which the interest is calcculated

Simple interest is directly proportional to pricipal, rate and time.

It means if the interest on a sum at a given rate of interest for 1 yr is Rs. 100, then interest for 2 yr, 3yr and 4 yr(other things being same) will be Rs.200, Rs. 300 and Rs. 400 respectivel)

Hence,

Amount=Principal+Simple interest

Simple Interest

1. P → Principle

   R → Rate

   T → Time

$$S.I. = \frac{P \times R \times T}{100} \text{ (yearly)}$$

2. Amount = P+S.I.

3. $S.I. = \dfrac{P \times R \times I}{100 \times 12}$ (monthly)

4. $S.I. = \dfrac{P \times R \times T}{100 \times 365}$ (Days)

5. When R = T

   Then $R = T = \sqrt{\dfrac{S.I.}{(Sum)} \times 100}$

6. Annual Instalment $= \dfrac{100 \times Amount}{100XT + R \times T(T-1)}$

Compound Interest

1. Amount $= P\left(1 + \dfrac{R}{100}\right)^n$

2. C.T. = A - P

3. C.T. $= P\left[\left(1 + \dfrac{R}{100}\right)^n - 1\right]$

4. (C.I. - S.I.) 2 years $= P\left(\dfrac{R}{100}\right)^2$

5. (C.I. - S.I.) 3 years

   $= P\left(\dfrac{R}{100}\right)^2 \left(\dfrac{300 + R}{100}\right)$

6. A sum of money becomes 3 times in 5 years at S.I. find R%

   $R = \dfrac{100(K-1)}{T}$

Example 1 Calculate the simple interest on Rs. 7200 at $12\frac{3}{4}\%$ per annum for 9 months.

Solution: $SI = \dfrac{P \times R \times T}{100} = \dfrac{7200 \times 51 \times 9}{4 \times 12 \times 100}$

$$\left(r = \frac{51}{4}\%, t = \frac{9}{12} yr\right) = Rs\ 688.50$$

Example 2 At what rate percent per annum will a sum of money double in 8yr?

Solution:

Let P = x, then A=2x, SI=(2x-x) = x

T = 8 years

$\therefore \quad R = \dfrac{SI \times 100}{P \times T} = \dfrac{x \times 100}{x \times 8} = 12.5\%$

Example 3 A certain sum of money amounts to Rs. 1008 in 2 yr and to Rs.1164 in $3\frac{1}{2}$ yr. find the sum and the rate of interest

Solution: SI for $\left(3\frac{1}{2} - 2\right) = 1\frac{1}{2} yr$

$= (1164-1008) = Rs.\ 156$

SI for 2 yr = Rs. 208

$\therefore$ Principal = Rs. (1008-208)=Rs. 800

Now, P=800, T=2yr and SI=Rs. 208

$\therefore \quad Rate = \dfrac{100 \times SI}{P \times T} = \dfrac{100 \times 208}{800 \times 2} = 13\%$

The difference in amount for two different time period is equal to the simple interest for the difference in two different time period.

Example 4 What annual instalment will discharge a debt of Rs. 1092 due in 3 yr at 12% simple interest?

Short cut the annual payment that will discharge a debt of Rs. A due in t years at r% rate of interest per annum is.

$$\frac{100A}{\left[100t + \dfrac{RT(T-1)}{2}\right]}$$

$$= \left(\frac{100 \times 1092}{100 \times 3 + \dfrac{3 \times 12(3-1)}{2}}\right) = \frac{109200}{336} = Rs.325$$

Compound Interest

It is the interest calculated on a sum of money which includes principal and interest calculated for previous year.

The SI and CI for first year is same and for second and subsequent years differ by an amount which is arrived by calculating interest on interest for previous years.

i. Simple interest for each year is constant

ii. Compound interest for each year includes- simple interest on principal and simple interest on interest calculated for previous year.

Let Principal= P, Rate=6% per anuum, time=n year and amount=A

I. When interest is compound annually

$$Amount = P\left(1 + \frac{R}{100}\right)^n$$

II. When interest is compound half-yearly

$$Amount = P\left(1 + \frac{R/2}{100}\right)^{2n}$$

III. When interest is compound quarterly

$$Amount = P\left(1 + \frac{R/4}{100}\right)^{4n}$$

IV. When interest is compound annually but time is in fraction, say $3\frac{2}{5}$ yr.

then, $\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$

V. Present worth of Rs. x due n years, hence is given by Present worth

$$= \frac{x}{\left(1+\frac{R}{100}\right)^n}$$

Example 5 *A sum of money at compound interest doubles itself 4yr. In how many years will it amount to eight times itself.*

Solution: Given, $2P = P\left(1+\frac{r}{100}\right)^4$

or $\left(1+\frac{r}{100}\right)^4 = 2$

$= \left[C.I. - S.I.\right]_{2year} = P\left(\frac{r}{100}\right)^2$

or $\left[\left(1+\frac{r}{100}\right)^4\right]^3 = (2)^3 = 8$

or $P\left(1+\frac{r}{100}\right)^{12} = 8P$

Hence required time is 12yr.

Example 6 On a certain sum of money, the simple interest for 2 yr is Rs. 50 at the rate of 5% per annum. Find the difference in CI and SI.

Solution: Using formula

Difference in CI and SI $= \frac{50\times 5}{200} = \text{Rs.}1.25$

Concept $50 = \frac{P\times 2\times 5}{100} \therefore P = \text{Rs.}500$

$A = 500 \times \frac{105\times 105}{100\times 100} = \text{Rs}\,551.25$

$\therefore$ CI=(551.25-500)= Rs. 51.25

Hence CI-SI=51.25-50= Rs. 1.25

When difference between the CI and SI on a certain sum of money for 2yr of r% rate is x, then the sum is given by.

$$\text{Sum} = \frac{\text{Difference}\times 100\times 100}{\text{Rate}\times\text{Rate}}$$

$$= \left[C.I. - S.I.\right]_{2year} = P\left(\frac{r}{100}\right)^2$$

Example 7 *The difference betwen the compound interestand the simple interest on a certain sum of money at 5% per annum for 2 yr is Rs. 1.50. Find the sum.*

Solution: Usint the above formula

$$\text{Sum} = 1.5\left(\frac{100}{5}\right)^2 = 1.5\times 400 = \text{Rs.}600$$

On a certain sum of money, the difference between compound interest and simple interest for 2 yr at r% rate is given by sum

$$\left(\frac{r}{100}\right)^2$$

On a certain sum of money, the difference between compound interest and simple interest for 3 yr of r% per annum is given by

$$\text{Difference} = \frac{\text{Sum}\times R^2\left(300+R\right)}{\left(100\right)^3}$$
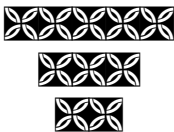
Exercise

1. Sudha borrowed Rs. 400 from her friend at the rate of 12% per annum for $2\frac{1}{2}$ yr. The interest and the amount paid by her were.

   (a) Rs. 140, Rs. 540
   (b) Rs. 130, Rs. 530
   (c) Rs. 125, Rs. 525
   (d) Rs. 120, Rs. 520

2. The simple interest on a sum of money is $\frac{1}{25}$ of the principal and the number of years is equal to the rate percent per annum. The rate percent is.

   (a) 3%   (b) 4%
   (c) 2%   (d) 2.5

3. If the simple interest on Rs. 1200 be more than the interest on Rs. 1000 by Rs. 30 in 3 yr. The rate percent per annum will be.

   (a) 5%   (b) 7%
   (c) 6%   (d) 8%

4. What annual payment will discharge a debt of Rs. 19350 due 4yr, hence at the rat of 5% simple interest?

   (a) Rs. 4500   (b) Rs. 5400
   (c) Rs. 4000   (d) None of these

5. Arun borrowed a sum of money from Jayant at the rate of 8% per annum simple interest for the first 4yr, 10% per annum for the next 6 yr and 12% per annum for the period beyond 10yr. If he pays a total of Rs. 12160 as interest only at the end of 15 yr, how much money did he borrow?

   (a) Rs. 8000   (b) Rs. 10000
   (c) Rs. 12000   (d) Rs. 9000

6. Anish borrowed Rs. 15000 at the rate of 12% and another amount at the rate of 15% for two years. The total interest paid by him was Rs. 9000. How

   (a) Rs. 1338   (b) Rs. 1188
   (c) Rs. 1378   (d) Rs. 1128

7. The simple interest on a sum of money will be Rs. 190 after 7 yr. In the next 7 yr principal becomes 3 times, what will be the total interest at the end of the 14th year?

   (a) Rs. 760   (b) Rs. 850
   (c) Rs. 750   (d) Rs. 780

8. The sum of money tyhat will amount to Rs. 5106 in $6\frac{1}{2}$ yr at $4\frac{1}{4}$ percent per annum at simple interest is.

   (a) Rs. 3600   (b) Rs. 4500
   (c) Rs. 4000   (d) Rs. 4400

9. The difference between the interest received from two different banks on Rs. 750 for 2 yr is Rs. 90. The difference between their rates is.

   (a) 4%   (b) 6%
   (c) 8%   (d) None of these

10. The simple interest on a sum of money will be Rs. 600 after 10yr. If the principal is trebled after 5 yr, the total interest at the end of 10 yr will be

    (a) Rs. 600   (b) Rs. 900
    (c) Rs. 1200
    (d) date inadequate

11. The rates of simple interest in two banks A and B are in the ratio 5 : 4. A person wants to deposit his total savings in two banks in such a way that he received equal half yearly interest from both. He should deposit the savings in banks A and B in the ratio.

(a) 2 : 5   (b) 4 : 5

(c) 5 : 2   (d) 5 : 4

12. A sum of money amount to Rs. 6690 after 3 yr and to Rs. 10035 after 6 yr on compount interest. The sum is

(a) Rs. 4460   (b) Rs. 3650

(c) Rs. 4535   (d) Rs. 2800

13. Anu borrows a sum of Rs. 1200 at the beginning of a year. After 4 months, Rs. 1800 is borrowed at a rate of interest double the previous one. At the end of the year, the sum of interest on both the loan is Rs. 216. What is the first rate of interest per annum?

(a) 9%   (b) 6%

(c) 8%   (d) 12%

14. Seema invested an amount of Rs. 16000 for two years at compound interest and received an amount of Rs. 17640 on maturity. What is the rate of interest?

(a) 8%   (b) 5%

(c) 4%   (d) 3%

15. What will be the compound interest acquired on a sum of Rs. 12000 for 3yr at the rate of 10% per annum?

(a) Rs. 2652   (b) Rs. 3972

(c) Rs. 3960   (d) Rs. 3852

16. A merchant commences with a certain capital and gains annully at the rate of 25%. At the end of 3yr his capital is worth Rs. 10000. What was his original capital?

(a) Rs. 5120   (b) Rs. 5220

(c) Rs. 5210   (d) Rs. 5130

17. What sum of money will amount to Rs. 699.66 in 2 yr, reckoning compound interest for 1 yr at 4% and for the other at $3\frac{1}{2}\%$ per annum?

(a) Rs. 560   (b) Rs. 650

(c) Rs. 670   (d) Rs. 580

18. The compound interest on Rs. 2000 at 5% per annum, compound yearly for $2\frac{1}{2}$ yr is.

(a) Rs. 260.12   (b) Rs. 214.25

(c) Rs. 250.25   (d) Rs. 210.10

19. If a sum of money at compound interest amount to thrice itself in 3yr, then in how many years will it be 9 times itself?

(a) 12yr   (b) 6yr

(c) 9 yr   (d) 15yr

20. At what rate percent will the compound interest, does a sum of money become 27 times in 3yr?

(a) 100%   (b) 150%

(c) 75%   (d) 200%

21. The compound interest interest on a certain sum for 2 yr is Rs. 105 and simple interest is Rs. 100. The rate of interest per annum and the sum are

(a) 2%, Rs. 1600   (b) 2%, Rs. 1400

(c) 3%, Rs. 1500   (d) 2%, Rs. 1500

22. On what sum will the difference between simple and compound interests for 3yr at 5% per annum amount to Rs. 12.20?

(a) Rs. 1600   (b) Rs. 800

(c) Rs. 1200   (d) Rs. 1500

23. Ravi gave Rs. 1200 on loan. Some amount he gave at 4% per annum simple interest and remaining at 5% per annum simple interest. After two years, he got Rs. 110 as interest. Then the amounts given at 4% and 5% per annum simple interest arek, respectively

(a) Rs. 500, Rs. 700

(b) Rs. 400, Rs. 800

(c) Rs. 800, Rs. 300

(d) Rs. 1100, Rs. 1100

24. The difference between compound interest and simple interest on a certain amount of money at 5% per annum for 2 yr is Rs. 15. Find the sum.

(a) Rs. 4500   (b) Rs. 7500

(c) Rs. 5000   (d) Rs. 6000

25. A owes B Rs. 1,573, payable $1\frac{1}{2}$ yr hence. Also B owes A Rs. 1,444.50, payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay whom and how much?

(a) A to A, Rs. 28.50

(b) B to A, Rs. 37.50

(c) A to B, Rs. 50

(d) B to A, Rs. 50

26. Consider the following statements.

If a sum of money is lent at simple interest, then the

I. money gets doubled in 5 yr if the rate of interest is $16\frac{2}{3}\%$

II. money gets doubled in 5yr if the rate of interest is 20%

III. money becomes four times in 10yr if it gets doubled in 5 yr.or these statements;

(a) I and III are correct

(b) II alone is correct

(c) III alone is correct

(d) II and III are correct

27. A man received Rs. 12000 as Puja Bonus. He invested a part of it at 5% per annum and the remaining at 6% per annum, simple interest being allowed in each case. The total interest earned by him in 4 years is Rs. 2580. The sum invested at 5% per annum is

(a) Rs. 4500   (b) Rs. 4000

(c) Rs. 7500   (d) Rs. 8000

28. A finance company declares that, at a certain compound interest rate, a sum of money deposited by anyone will become 8 times in three years. If the same amount is deposited at the same compound rate of interest, then in how many years will it ecome 16 times?

(a) 5 yr   (b) 4 yr

(c) 6 yr   (d) 7 yr

29. Seema invested an amount of Rs. 16,000 for two years on compound interest and received an amount of Rs. 17,640 on maturity.

What is the rate of interest?

(a) 5 pcpa   (b) 8 pcpa

(c) 4 pcpa

(d) Data inadequate

Solution

1. **(d)** $SI = \dfrac{400 \times 5 \times 12}{2 \times 100} = Rs.120$

$A = (400+120) = Rs.\ 520$

2. **(c)** $\dfrac{1}{25}P = \dfrac{P \times R \times R}{100}$ or, $R^2 = 4 \therefore R = 2\%$

3. **(a)** $\dfrac{1200 \times 3 \times R}{100} - \dfrac{100 \times 3 \times R}{100} = 30$

4. **(a)** Let the annual instalment be Rs x.

Then, 4x+{interest on x for (3+2+1) years} = 19350

$\Rightarrow 4x + \dfrac{x \times 6 \times 5}{100} = 19350$

$\therefore x = Rs.\ 4500$

5. **(a)** Interest for 4yr@ 8%+interest for 6 yr@10%+interest for 5 yr@12%

$= 32+60+60 = 152\%$

Then, $12160 = \dfrac{P \times 152}{100} \Rightarrow P = Rs.8000$

6. **(b)** $\dfrac{15000 \times 12 \times 2}{100} + \dfrac{P \times 15 \times 2}{100} = 9000$

or $\dfrac{P \times 30}{100} = (9000-3600) = 5400$

or $\qquad P = Rs.\ 1800$

$\therefore \qquad$ Total sum borrowed

$= Rs.\ (1800+15000) = Rs.33000$

7. **(a)** Simple interest for the next 7 yr

$= 3 \times 190 = Rs/\ 570$

Therefor, simple interest after14yr

$= (190+570) = Rs.\ 760$

8. **(c)** $(5105-P) = \dfrac{P \times 17 \times 13}{4 \times 2 \times 100}$

$5105-P = P \times 0.27625$

$\therefore$

$P = \dfrac{5105}{1.27625} = Rs.4000$

9. **(b)** $\dfrac{750 \times 2}{100}(R_1 - R_2) = 90$

$\Rightarrow (R_1 - R_2) = \dfrac{90 \times 100}{750 \times 2} = 6\%$

10. **(c)** If the simple interest for 10 yr is Rs. 600, then simple interest for 5 yr will be Rs. 300. Now since the principal is trebled after 5 yr. The interest will also be trebled for the next 5 yr ie, 900.

Therefore, total interest as the end of 10yr $= (300+900)$

$= Rs.\ 1200$

11. **(b)** Rate of interest $= 5x$ and $4x$

Let he invests Rs. P in bank A and Q in bank B

Then, $\dfrac{P \times 5x \times 1}{2 \times 100} = \dfrac{Q \times 4x \times 1}{2 \times 100}$

$5P = 4Q$ or $P : Q = 4 : 5$

12. **(a)** Let the sum be Rs. P.

Then, $P\left(1+\dfrac{R}{100}\right)^3 = 6690$ ...(i)

and $P\left(1+\dfrac{R}{100}\right)^6 = 10035$ ...(ii)

On dividing, we get

$$\left(1+\frac{R}{100}\right)3=\frac{10035}{6690}=\frac{3}{2}$$

From Eq. (i)

$$P\times\frac{3}{2}=6690\,or\,P=Rs.4460$$

13. (b) Let the rate of interest be R%

Then, interest earned from
Rs. 1200 at the end of the year
= 1200 R/100 = Rs. 12R

Again, interest earned from Rs.
1800 at the end of the year
=(1800/100)×(8/12)×2R=Rs. 24R

So, total interest earned=36R,
which equals 216

$$\Rightarrow\quad R=\frac{216}{36}=6\%$$

14. (b) $$\frac{A}{P}=\left(1+\frac{R}{100}\right)^{n}=\frac{17640}{16000}=\left(1+\frac{R}{100}\right)^{2}$$

$$\Rightarrow\frac{17640}{16000}=\left(1+\frac{R}{100}\right)^{2}\Rightarrow\left(\frac{21}{20}\right)^{2}$$

or $$\frac{1+R}{100}=\frac{21}{20}$$

$$\therefore\qquad R=5\%$$

15. (b) $$CI=12000\left(1+\frac{10}{100}\right)^{3}-12000$$

$$= 15972-12000 = Rs.\ 3972$$

16. (a) Let his initial capital be Rs. x.

Then, $$x\times\frac{125}{100}\times\frac{125}{100}\times\frac{125}{100}=10000$$

$$x\times\left(\frac{5}{4}\right)^{3}=10000$$

$$\therefore\qquad x = Rs.\ 5120$$

17. (b) $$699.66=\frac{P\times104\times207}{100\times200}$$

$$\therefore\ P=\frac{699.66\times100\times200}{104\times200}=Rs\,650$$

18. (a) $$A=P\left(1+\frac{R}{100}\right)^{2}\times\left(1+\frac{R/2}{100}\right)$$

$$=2000\left(\frac{105}{100}\right)^{2}\times\left(\frac{205}{200}\right)$$

$$=\frac{2000\times105\times105\times205}{100\times100\times200}$$

= Rs. 2260.12

$$\therefore\ CI=Rs\ (2260.12-2000)$$

=Rs.260.12

19. (b) It will become (3)2 times
in 2×3=6yr

20. (d) $$(3)^{3}=\left(1+\frac{R}{100}\right)^{2}\Rightarrow\frac{R}{100}=3-1=2$$

$$\therefore\ R=200\%$$

21. (a) The difference between CI and SI
for 2 yr is the interest on the first
year's interest.

SI for the first year $$=\frac{100}{2}=Rs.50$$

CI-SI =105-100 =Rs. 5

or $$5=\frac{50\times1\times R}{100}\,or\,r=10\%$$

Again, $SI = \dfrac{P \times R \times R}{100} \Rightarrow 100 = \dfrac{P \times 10 \times 2}{100}$

or P=Rs. 500

22. (a) If the difference between CI and SI on a certain sum at r% per annum for 3 yr is given, then sum is given by.

$$Sum = \dfrac{Difference \times (100)^3}{r^2 (300+r)}$$

23. (a) Let the amount given at 4% per annum be Rs. x.

$\therefore$ amount given at 5% per annum = Rs. (1200-x)

$\dfrac{x \times 4 \times 2}{100} + \dfrac{(1200-x) \times 5 \times 2}{100} = 100$

$\Rightarrow$ x=500

Also, (1200-x)=1200-500=700

24. (d) Let the sum of Rs. 100

then $SI = \dfrac{100 \times 5 \times 2}{100} = 10$

and $CI = 100\left(1 + \dfrac{5}{100} - 100\right)$

$= 100 \times \dfrac{21 \times 21}{20 \times 20} - 100 = \dfrac{41}{4}$

$CI - SI = \dfrac{41}{4} - 10 = \dfrac{1}{4}$

If the difference is $\dfrac{1}{4}$ the sum-100

If the difference is Rs. 15, then sum $= 400 \times 15 =$ Rs. 6000.

25. (d) Let the present value of what A owes

B Rs. x.

Then, $x + \dfrac{x \times 14 \times 3}{2 \times 100} = 1573$

or $x + \dfrac{21}{100}x = 1573$ or $\dfrac{121x}{100} = 1573$

or

$x = \dfrac{1573 \times 100}{121} = Rs.1300$

Let be the present value of what B owes A.

$\Rightarrow$ $y + y \times \dfrac{1}{2} \times \dfrac{14}{100} = Rs.1444.50$

$\Rightarrow$ $y + \dfrac{7}{100}y = Rs.1444.50$

$\Rightarrow$ $y = \dfrac{1444.50 \times 100}{107} = Rs.1350$

Hence, B must pay Rs. 50 to A.

26. (b) Let the sum of money lent be Rs.P Then,

I.SI received after 5yr $= \dfrac{P \times 50 \times 5}{3 \times 100}$

$= \dfrac{5}{6}P$

Amount after 5yr $= \dfrac{P \times 20 \times 5}{100} = P$

II.SI after 5yr $= \dfrac{P \times 20 \times 5}{100} = P$

Amount after 5yr=P+P=2P

Therefore, II is correct.

III. Amount after 10yr at the rate of 20%

$$= P + \frac{P \times 20 \times 10}{100} = P + 2P = 3P$$

(At 20% money gets doubled in 5yr)
Therefore, II is not correct
Hence, (b) is the correct option.

27. (c)

28. (b) 8 time in 3yr, this means it doubles every year. So it will become 16 times the next year.

29. (a) Differece = 1640. So the gets aprox 8000 interest for one year. Which is 5%

30. (c) Let the pricipal be Rs. 100.

Then, $SI = \frac{100 \times 20 \times 3}{100} = Rs. 60$

$$CI = 100\left(1 + \frac{20}{100}\right)^3 - 100$$

$$= 100 \times \left(\frac{6}{5}\right)^3 - 100 = \frac{364}{5}$$

$$CI\text{-}SI = \frac{364}{5} - 60 = \frac{64}{5}$$

If difference is Rs. $\frac{64}{5}$,

Principal = Rs. 100
If difference is Rs. 48, Principal

$$= \frac{100 \times 5}{64} \times 48 = Rs. 375$$
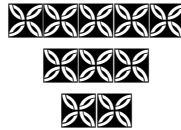
Time and work is another important section of quantitative apritude where conceptual clarity of relationship between working efficiency and time is very important to understand.

Working efficiency is the work done by an individual in one day and this efficiency is inversely proportional to the number of days to complete a work. It means that a person who takes less days to complete a work is said to be more efficient than a man who takes more days to complete the same work.

❑   Working efficiency is directly proporational to the number of persons employed to complete the work.

❑   Working efficiency is inversely proportional to the number of days taken to complete a work.

Hence if A does one work in 10 days and B does the same work in 20 days, then A is said to be twice as efficient as B.

These results can be used to find quick solutions of some important questions.

If m_1, persons can do w_1 working d_1 days and m_2, persons can do w_2 works in d_2 days then we have a very general formula in the relationship on $m_1 d_1 w_2 = m_2 d_2 w_1$

Example 1   40 men can cut 60 trees in 8h. If 8 men leave the job, how many trees will be cut by 32 men in 12h?

Short cut method

$$m_1 \times d_1 \times w_2 = m_2 \times d_2 \times w_1$$

$$40 \times 8 \times w_2 = 32 \times 12 \times 60$$

$$\therefore \qquad w_2 = \frac{32 \times 12 \times 60}{40 \times 8} = 72 \text{trees}$$

If m_1 persons can do w_1 works d_1 days working t_1 hours a day and m_2 persons can do w_2 worksinh d_2 days working t_2 hours a day then we have.

$$m_1 \times d_1 \times t_1 \times w_2 = m_2 \times d_2 \times t_2 \times w_1$$

If A can do a piece of work in x days and B cando it in y days then A and B working together will do the some work in $\left(\dfrac{xy}{x+y} \right)$ days.

Example 2   A can do piece of work in 30 days, while B can do it in 40 days. A and B working together can do it in how many days?

Using Short Cut Formula

Required number of days

$$= \left(\frac{30 \times 40}{30 + 40} \right) = 17\frac{1}{7} \text{days}$$

If A, B and C can do a work in x, y and z days respectively then all of them together can finish the work in

$$\left(\frac{xyz}{xy + yz + xz} \right) \text{days}$$

Example 3: A takes twice as much time as B and thrice as much time as C to finish a piece of work, working together they can finish the work in 2 days, find the time each will take to finish the work.

Using Short Cut Formula

$$\frac{a \times \dfrac{a}{2} \times \dfrac{a}{3}}{a \times \dfrac{a}{2} + \dfrac{a}{2} + \dfrac{a}{3} + a \times \dfrac{a}{3}} = 2\text{days(Given)}$$

or, $\dfrac{\dfrac{a^3}{6}}{\dfrac{a^2}{2}+\dfrac{a^2}{6}+\dfrac{a^2}{3}}=2 \Rightarrow \dfrac{a^3}{6}\times\dfrac{6}{6a^2}=2\, or\, a=12$

or, $a=12, b=6, c=4$

If A and B can do a piece of work in x days, B and C in y days, C in y days, then (A+B+C) working together will do the same work in

$$=\left[\dfrac{2xyz}{xy+yz+xz}\right] days$$

Example 4: A job is completed by 10 men in 20 days and by 20 women in 15 days. How many days will it take for 5 men and 10 women to finish that work?

Solution: Let 1 man's 1 day's work= x and 1 woman's 1 day's work=y.

Then, $10x=\dfrac{1}{20}$ and $20y=\dfrac{1}{15}$

So, $x=\dfrac{1}{200}$ and $y=\dfrac{1}{300}$

$\therefore$ (5men+10 women)'s day's work

$=\dfrac{5}{200}+\dfrac{10}{300}=\dfrac{7}{120}$

Therefore, required number of

days $=17\dfrac{1}{7}$ days.

Example 5 A sum of money is sufficient to pay A's wages for 21 days and B's wages for 28 days. The some money is sufficient to pay the wages of both for how many days?

Solution: Let total money be Rs. x.

A's one day wages $=\dfrac{x}{21}$

B's one day wages $=\dfrac{x}{28}$

$\therefore$ (A+B)'s one day wages $=\dfrac{x}{12}$

Therefore, money is sufficient to pay the wages for 12 days.

Wages are distributed in the ratio of working capacity.

Example 6 There is a leak in the bottom of the tank. This leak can empty a full tank in 8h. When the tank is full, a tap is opened into the tank which intakes water at a rate of 6L/h and the tank is now emptied in 12h. What is the capacity of the tank?

Solution: In 1h the leak empties $=\dfrac{1}{8}$ tank.

Now let the tap fill the tank in x hours then

$$\dfrac{1}{x}=\dfrac{1}{8}-\dfrac{1}{12}=\dfrac{1}{24}$$

Therefore, the tap fills the tank in 24 h.

Therefore, capacity of the tank=$6\times24=144$L

Example 7 A, B and C contract a work for Rs. 550. Together A and B are supposed to do $\dfrac{7}{11}$ work.

How much does C get?

Solution: A and B are supposed to do $\dfrac{7}{11}$ work. Then C is supposed to do the remaining

work $\left(1-\dfrac{7}{11}\right)=\dfrac{4}{11}$ work

Ratio of working capacity

$$=(A+B):C=\frac{7}{11}:\frac{4}{11}=7:4$$

$$\therefore \quad \text{C's share} = \frac{550}{11}\times 4 = \text{Rs.200}$$

Example 8 A cistern is filled by pipe A in 10h and the full cistern can be leaked out by an

exhaust pipe B in 12 h. If both the pipes are opened in what time the cistern is full?

Solution: Work done by both the pipes in

$$1 \text{ hour} = =\left(\frac{1}{10}-\frac{1}{12}\right)=\frac{1}{60}$$

$\therefore$ The cistern will be full in 60h.

Exercise

1. A company has a job to prepare certain number cans and there are three machines A, B and C for this job. A can complete the job in 3 days, B can complete the job in 4 days and C can complete the job in 6 days. How many days the company will take to complete the job if all the machines are uses simultaneously?

   (a) 4 days   (b) $\frac{4}{3}$ days

   (c) 3 days   (d) 12 days

2. A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

   (a) 20 days   (b) 25 days
   (c) 23 days   (d) 27 days

3. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

   (a) $12\frac{2}{3}$ days   (b) 24 days

   (c) 14 days   (d) $12\frac{1}{2}$ days

4. A and B can do a piece of work in 72 days; B and C can do it in 120 days; A and C can do it 90 days. In what time can A alone do it?

   (a) 150 days   (b) 120 days
   (c) 8 days   (d) 9 days

5. X takes 4 days to complete one-third of a job, Y takes 3 days to complete one-sixth of the same work and Z takes 5 days to complete half the job. Ifall of them work together for 3 days and X and Z quit, how long will it take for Y to complete theremaining work done?

   (a) 6 days   (b) 8.1 days
   (c) 5.1 days   (d) 7 days

6. Two men, A and B, started a job in which A was thrice as good as B and therefore took 60 days less than B to finish the job. How many days will they take to finish the job, if they start working together?

   (a) 20 days   (b) $22\frac{1}{2}$ days

   (c) 25 days   (d) 30 days

7. The work done by a man, a woman and a child are in the ratio 3 2 1. If daily wages of 20 men, 30 women and 36 children amount to Rs. 78. What will be the wages of 15 men, 21 women and 30 children for 18 weeks?

   (a) Rs. 7371   (b) Rs. 8645

   (c) Rs. 9000   (d) None of these

8. Three men earn as much as 4 women, 4 women earn

   (a) Rs. 115   (b) Rs. 135

   (c) Rs. 125   (d) Rs. 150

9. A and B can do a job in 10 days, B and C can do the same job in15 days. If all three together can do the work in 6 days, then in how many days can B complete the whole job?

   (a) 10 days   (b) 12 days

   (c) 18 days   (d) none of these

10. 20 women can do a job in 20 days. After each day, one woman is replaced by a man or a boy alternatively starting with a man. Man is twice efficient and boy is half efficient as a woman. On which day does the job get completed?

   (a) 16th day   (b) $\dfrac{50}{3}$ day

   (c) 22 day   (d) 20 day

11. A does half as much work as B in three-fourth of the time. If together they take 18 days to complete the work, how much time shall B take to do it?

   (a) 30 days   (b) 35 days

   (c) 40 days   (d) none of these

12. A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. B alone can do the whole in

   (a) 12 days   (b) 15 days

   (c) 20 days   (d) 30 days

13. A, B and C can do a piece of work in 11 days, 20 days and 55 days respectively, working alone. How soon can the work be done if A is assisted by B and C on alternate days

   (a) 7 days   (b) 8 days

   (c) 9 days   (d) 10 days

14. Ram can do a piece of work in 8 days which Shyam can finish in 12 days. If they work at it on alternate days with Ram beginning, in how many days, the work will be finished?

   (a) $9\dfrac{1}{3}$ (b) $9\dfrac{1}{2}$

   (c) $9\dfrac{1}{24}$ (d) $10\dfrac{1}{3}$

15. 12 Children take 16 days to complete a work which can be completed by 8 adults in 12 days. 16 adults started working and after 3 days 10adults left and 4 children joined them. How many days will it take them to complete the remaining work?

   (a) 6   (b) 8

   (c) 4   (d) 3

16. 12 men can complete a work in 18 days. Six days after they started working, 4 men joined them. How many days will all of them take of finish the remaining work?

   (a) 9   (b) 10

   (c) 12   (d) 15

17. 8 men can dig a pit in 20 days. If a man works half as much again asa boy, then

4 man and 9 boys can digasimilar pit in.

(a) 10 days   (b) 12 days

(c) 15 days   (d) 16 days

18. Ramesh can finish a job in 20 days. He worked for 10 days alone and completed the remaining job working with Dinesh in 2 days, How many days would both Dinesh and Ramesh together take to complete the entire job?

(a) 4   (b) 5

(c) 10   (d) 12

19. A worker is paid Rs. 56 for 35 hour week. Upto 40h, he is paid at the normal rate and on overtime, 1.5 times the normal. How many hours did he work to get Rs. 88?

(a) 48   (b) 50

(c) 58   (d) 55

20. A and B weave a carpet in 10 days and 15 days respectively. They begin to work together but B leaves after 2 days. In what time will A complete the remaining work?

(a) $6\frac{1}{3}$ days   (b) $6\frac{2}{3}$ days

(c) 7 days   (d) 8 days

21. A cistern is filled in 5h and it takes 6h when there is a leak in its bottom. If the cistern is full in what time shall the leak empty it.

(a) 6h   (b) 5h

(c) 30h   (d) 15h

22. A pipe can fill a cistern in 6 hours. Due to a leak in its bottom it is filled in 7 hours. When the cistern is full, in how much time will it be emptied by the leak?

(a) 42h   (b) 40h

(c) 43h   (d) 45h

23. Two men and 7 children complete a certain piece of work in 4 days while 4 men and 4 children complete the same work in only 3 days. The number of days required by 1 man to complete the work is.

(a) 60 days   (b) 15 days

(c) 6 days   (d) 51 days

24. A tap can fill a tank in 16 min and another can empty it in 8 min. If the tank is already 1/2 full and both the taps are opened together, will the tank be filled or emptied? How long will it take before the tank is either filled or emptied completely as the case may be

(a) Emptied; 16 min

(b) Filled; 8 min

(c) Emptied; 8min

(d) Filled;12 min

25. A can do a piece of work in 10 days, while B alone can do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get Rs. 450 for the whole work, how should they divide the money?

(a) Rs. 225, Rs. 150, Rs. 75

(b) Rs. 250, Rs. 100, Rs. 100

(c) Rs. 200, Rs. 150, Rs. 100

(d) Rs. 175, Rs. 175, Rs. 100

26. A rectangular tank is 225 m by 162 m at the base. With what speed must water flow into it through an aperture 60 cm by 45 cm that the level may be raised 20 cm in 5h?

(a) 5000 m/h (b) 5400 m/h

(c) 5200 m/h (d) 5600 m/h

27. Two taps can fill a tank in 15 and 12 min, respectively. A third tap can empty it in 20 min. If all the taps are opened at the same time, then in how much time will the tank be filled?

(a) $8\dfrac{1}{2}$ min (b) 10 min

(c) $11\dfrac{1}{3}$ min (d) 14 min

28. C is twice efficient as A. B takes thrice as many days a C. A takes 12 days to finish the work alone. If they work in pairs (i.e., AB, BC, CA) starting with AB on the first day, BC on the second day and AC on the third day third day and so on, then how many days are required to finish the work?

(a) $6\dfrac{1}{5}$ days (b) 4.5 days

(c) $5\dfrac{1}{9}$ days (d) 8 day

29. A team of workers was employed by a contractor who undertook to finish 360 pipeces of an article in a certain number of days, Making four more pieces per day than was planned, they could complete the job a day ahead of schedule.

How many days did they take to complete the job?

(a) 10 days (b) 8 days

(c) 9 days (d) 12 days

30. Three pipes are made of different shapes. The corss-sections of the pipes are an equilateral triangle, a hexagone and a circle. The perimeter of each of these cross-sections is equal. The flow through the pipes is proportional to the area of cross-section. If it takes 8 min for the tringular pipe to fill up the tank, what will be the difference in the times taken by the hexagonal and circular pipes?

(a) 45s (b) 1 min

(c) 1.5 (d) 7.9 min
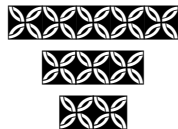
Solution

1. (b) Work done by all the machines working together.

$$= \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right) = \frac{4+3+2}{12}$$

$$= \frac{9}{12} = \frac{3}{4} \text{ work}$$

Hence, required number of days to complete the job $= \frac{4}{3}$ days.

2. (d) Let A does one work in x days

then B will do the same work in 2x days.

then $\frac{1}{x} + \frac{1}{2x} = \frac{1}{18} \Rightarrow x = 27$ days.

3. (d) (2M+3B)s 1 day work $= \frac{1}{10}$

(3M+2B)s 1 day work $= \frac{1}{8}$

$$\Rightarrow \quad M = \frac{7}{200}, \quad B = \frac{1}{100}$$

$\therefore$ (2M+1B)s 1day's work

$$= \left(2x \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$$

Hence, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ day

4. (b) (A+B+C) working together complete the work in.

$$\frac{2 \times 72 \times 90 \times 120}{72 \times 120 + 120 \times 90 + 72 \times 90} \text{ days}$$

$= 60$ days

A alone will complete the work in

$$\frac{60 \times 120}{120 - 60} = 120 \text{ days.}$$

5. (c) Work done by all of three in 3 days

$$= 3\left(\frac{1}{12} + \frac{1}{18} + \frac{1}{10}\right) = \frac{43}{60} \text{ work left}$$

$$= \frac{17}{60} \text{ required}$$

number of days taken by Y to complete the remaining work

$$= \frac{17}{60} \times 18 = 5.1 \text{ days}$$

6. (b) Let's assume that A takes x days to finish the job then B will take 3x days.

$3x - x = 60 \Rightarrow x = 30$ days

Work done by (A+B) in ond day

$$= \left(\frac{1}{30} + \frac{1}{90}\right) = \frac{2}{45}$$

Hence they together will do the job in $\frac{45}{2}$ days $= 22\frac{1}{2}$ days.

7. (a) $\frac{1}{3}M = \frac{1}{2}W = 1C \Rightarrow 2M = 3W = 6C$

20M+30W+36C=60C+60C+36C

$= 156C$

If 156 children get Rs. 78, 1 child

gets $\text{Re} \dfrac{78}{156}$

Now, $15M + 21W + 30C = 45C$

$+42C + 30C = 117C$

$\therefore$ 117 children should get

per

day $\text{Rs} \left(\dfrac{78}{156} \times 117 \right)$ per

day

$\therefore$ For (18×7) days, they

should get.

$$\text{Rs.}(18 \times 7)\left[\dfrac{78}{156} \times 117 \right] = \text{Rs.}7371.$$

8. (c) Earning of one girl = Rs. 50

Earning of 10 girl = Rs. 500

Earning of 8 boys = Rs. 500

Earning of 6 boys $= \text{Rs.} \dfrac{6}{8} \times 500$

= Rs. 375

$\therefore$ Earning of 4 women=

Rs. 375

Earning of 3 men = Rs.

375

Earning of 1 men = Rs.

125

9. (d) (A+B+C)s ond day work$= \dfrac{1}{6}$ ...(i)

(A+B)s one day work$= \dfrac{1}{10}$ ...(ii)

(B+C)s one day work$= \dfrac{1}{15}$ ...(iii)

$\therefore$ (A+C)s ond day work

$$\left(\dfrac{1}{6} - \dfrac{1}{10} \right) + \left(\dfrac{1}{6} - \dfrac{1}{15} \right) = \dfrac{1}{6}$$

Since A and C together can complete

$\dfrac{1}{6}$ work in one day which is also the

working capacity ofn all the three

persons. Hence B's share in the total

work is nil.

10. (b) Let the total work be $20 \times 20 = 400$ unit

each woman does 1 unit per day.

each man does 2 unit per day, while

each boy does $\dfrac{1}{2}$ unit per day

So, work done on first days = 20 unity

Work done on Sencond days= 21 unit

Work done on third days = 20.5 unit

Work done in 4th days = 21.5 unit

(First day+Second day) + (third day+

Fourth day) +...=41+42+43

$+ ...+49 = 405 > 400$

There are 9 such pairs of days.

Therefore, the work will be completed

on the 18th day.

11. (a) Let B takes x days to do the work.

Then, A takes $\left(2 \times \dfrac{3}{4}x \right) = \dfrac{3x}{2}$

days to do it.

(A+B)s one day work $= \dfrac{1}{18}$

or,

$$\dfrac{1}{x} + \dfrac{2}{3x} = \dfrac{1}{18} \Rightarrow x = 30$$

12. (b) Let B takes x days to complete a

work.

Then A will take $\dfrac{x}{3}$ days to complete the same work.

Given $\left(x - \dfrac{x}{3}\right) = 10 \Rightarrow x = 15\,\text{days}$

13. (b) (A+B)s one day work

$= \left(\dfrac{1}{11} + \dfrac{1}{20}\right) = \dfrac{31}{220}$

(A+C)s one day work

$= \left(\dfrac{1}{11} + \dfrac{1}{55}\right) = \dfrac{6}{55}$

Work done in 2 days

$= \dfrac{31}{220} + \dfrac{6}{55} = \dfrac{55}{220} = \dfrac{1}{4}$

Now, $\dfrac{1}{4}$ work is done in 2 days.

∴ Whole work will be done in $(4 \times 2) = 8$ days.

14. (b) Work done in 2 days

$= \left(\dfrac{1}{8} + \dfrac{1}{12}\right) = \dfrac{5}{24}$

Work done in 8 days $= \dfrac{5}{6}$

Work done in 9 days $= \dfrac{5}{6} + \dfrac{1}{8} = \dfrac{23}{24}$

Remaining work $= \dfrac{1}{24}$

Now, it is Syam's turn on 10th days.

$\dfrac{1}{12}$ work is done by him in one day

$\dfrac{1}{24}$ work will be done by him in

$12 \times \dfrac{1}{24} = \dfrac{1}{2}\,\text{days.}$

∴ Total time taken $= 9\dfrac{1}{2}\,\text{days.}$

15. (a) Let 1 child's 1 day work = x and 1 adult's 1 day's work=y

Then, $12x = \dfrac{1}{16}$ or $x = \dfrac{1}{192}$

and $8y = \dfrac{1}{12}$ or $y = \dfrac{1}{96}$

Work done in 3 days

$= 16 \times \dfrac{1}{96} \times 3 = \dfrac{1}{2}$

Remaining work $= \dfrac{1}{2}$

(6 adults+children)'s 1 day's work

$= \dfrac{6}{96} + \dfrac{4}{192} = \dfrac{1}{12}$

$\dfrac{1}{12}$ work is done by them in 1 day.

$\dfrac{1}{2}$ work is done by them in

$12 \times \dfrac{1}{2} = 6$ days.

16. (a) 12 men's 6 days work $= \dfrac{1}{3}$ Remaining work $= \dfrac{2}{3}$

$m_1 \times d_1 \times w_2 = m_2 \times d_2 \times w_1$

$12 \times 1 \times \dfrac{2}{3} = 16 \times d_2 \times \dfrac{1}{18}$

$$\Rightarrow \qquad d_2 = \frac{12 \times 2 \times 18}{3 \times 16} = 9 \text{ days}$$

17. (d) $1 \text{ men} = 1 + \dfrac{1}{2} = \dfrac{3}{2} \text{ boys}$

$\therefore \qquad 4 \qquad \text{men} + 9 \text{boys} =$

$= \dfrac{4 \times 3}{2} + 9$

= 15 boys and 8 men

$= 8 \times \dfrac{3}{2} = 12 \text{ boys}$

Now $m_1 \times d_1 = m_2 \times d_2$

$d_2 = \dfrac{m_1 \times d_1}{m_2} = \dfrac{12 \times 20}{15} = 16 \text{ days}$

18. (a) Ramesh alone finished $\dfrac{1}{2}$ of the work in 10 days. Remaining $\dfrac{1}{2}$ of the job was finish by Ramesh and Dinesh together in 2 days. Therefore, they both together can finish complete job in 4 days.

19. (b) Let the worker for $(40+x)$ hours, then $\dfrac{56}{35} \times 40 + \dfrac{x \times 15 \times 56}{35} = 88$

2.4x=24

$\Rightarrow \qquad x=10$

20. (b) (A+B)'s one day work $= \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{1}{6}$

(A+B)'s two days work $= \dfrac{2}{6} = \dfrac{1}{3}$

or $\qquad \dfrac{x+8}{x} = \dfrac{60}{50} \Rightarrow x = 40$

Hence, there were 40 men, originally

21. (c) Let the leak empties the tank in x

hours.

Then, $\qquad \dfrac{1}{5} - \dfrac{1}{x} = \dfrac{1}{6}$

$\dfrac{1}{x} = \dfrac{1}{5} - \dfrac{1}{6} = \dfrac{1}{30}$

$\therefore$ The leak will empty the tank in 30h.

22. (a) Resultant work done by both the pipes in one hour.

$= \left(\dfrac{1}{6} - \dfrac{1}{7} \right) = \dfrac{1}{42} \text{ of cistern}$

$\therefore$ The whole cistern will be emptied in 42 h

23. (b) (2M+7C)'s 1 day work $= \dfrac{1}{4}$

It means that 1 work will be finished by (8M+28C)

Again (4M+4C)'s day work $= \dfrac{1}{3}$

or 1 work will be completed by 12M+12C.

$\therefore \qquad 8M+28C=12M+12C$

$\Rightarrow \qquad M=4C$

$\therefore \qquad 4M+4C=5M$

24. (c) If both the pump are working together then the tank will be emptied because the work efficiency of pump empting is more than that of pump filling it. Hence in 1 min net work done

$= \left(\dfrac{1}{8} - \dfrac{1}{16} \right) = \dfrac{1}{16} \text{ tanks.}$

or 1 tank will be emptied in 16 min

or $\dfrac{1}{2}$ tank will be emptied in 8 min

25. (a) Work done by A and B in 5 days

$= \left(\dfrac{1}{10} + \dfrac{1}{15} \right) \times 5 = \dfrac{5}{6}$

Remaining work $= \left(1 - \dfrac{5}{6}\right) = \dfrac{1}{6}$

$\therefore$ C alone can do the work in

$6 \times 2 = 12$ days

Ratio of their work

$= \dfrac{5}{10} : \dfrac{5}{15} : \dfrac{2}{12} = 3 : 2 : 1$

26. (b) Required rate

$\dfrac{225 \times 162 \times 20}{5 \times 100} = \dfrac{60}{100} \times \dfrac{45}{100} \times h$

$\Rightarrow$ h=5,400 m/h

27. (b) When all the three taps are opened at the same time, the part of the tank filled in one minute is.

$\dfrac{1}{15} + \dfrac{1}{12} - \dfrac{1}{20} = \dfrac{4 + 5 - 3}{60} = \dfrac{6}{60}$

Tank will be completely filled in

$\dfrac{60}{6} = 10\,\text{min}.$

28. (c)

	A	B	C
Efficiency	3 :	2 :	6
No. of days	2 :	3 :	1

$\Rightarrow$ Number of days taken by A=12, Number of days taken by B=18 and Number of days taken by C=6

1 day's work of (A+B) $= \dfrac{5}{36}$

1 day's work of (B+C) $= \dfrac{8}{36}$

1 day's work of (C+A) $= \dfrac{9}{36}$

In 5 days total work done $= \dfrac{35}{36}$

Now, the rest of the work $\left(ie, \dfrac{1}{36}\right)$ is done by AC

Number of days taken by AC for the rest of the work $= \dfrac{1/36}{9/36} = \dfrac{1}{9}$

Therefore, total time taken to complete the work $= 5 + \dfrac{1}{9} = 5\dfrac{1}{9}$ days.

29. (a) Let x be the usual rate of work.

Then, $\dfrac{360}{x+4} - \dfrac{360}{x} = 1$

On solving,

We get x=36 and the number of

days $\dfrac{360}{36} = 10$

30. (b)
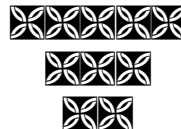
RATIO

If a and b are two quantities of the same kind, then $\dfrac{a}{b}$ is known as the ratio of a and b. Therefore, the ratio of two quantities in the same units is a fraction that one quantity is of the other.

Thus, a to b is a ratio $\dfrac{a}{b}$, written as a:b

The firstterm of the ratio is called **antecedent,** while the second term is called **consequent.**

Ratio between 30 kg and 50kg is 3:5

The multiplication or division of each term of a ratio by a same non-zero number does not effect the ratio. Hence 3:5 is the same as 6 : 10 or 9 : 15 or 12 : 20 etc. Ratio can be expressed as percentages. To expressthe value of a ratio as a percentage, we multiply the ratio by 100

Therefore, $\dfrac{3}{5} = 0.6 = 60\%$

RATIO & PROPORTION

1. $\dfrac{a}{b} = \text{Ratio} = \dfrac{N(\text{antesedent})}{D(\text{Consequent})}$

2. $\dfrac{a^2}{b^2} = \text{Duplicate ratio}$

3. $\dfrac{a^3}{b^3} = \text{Triplicate Ratio}$

4. $\dfrac{\sqrt{a}}{\sqrt{b}} = \text{Sub Duplicate Ratio}$

5. $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \text{Sub Triplicate Ratio}$

6. $a: b:: c: d \rightarrow \dfrac{a}{b} = \dfrac{c}{d} \left(\dfrac{\text{Third Pr opration}}{\text{Fourth Pr opration}} \right)$

7. Mean Propration for two number

   a and b $= \sqrt{ab}$

8. Third Proportion for two numbers

   a and b $= \dfrac{b^2}{a}$

9. To distribute Rs. x among a : b : c

   $'a' = \left(\dfrac{a}{a+b+c} \right) \times x$

   $'b' = \left(\dfrac{b}{a+b+c} \right) \times x$

   $'c' = \left(\dfrac{c}{a+b+c} \right) \times x$

10. If $\dfrac{a}{b} = \dfrac{5}{3}, \dfrac{b}{c} = \dfrac{7}{4}$ find a : b : c

    $\dfrac{a}{b} = \dfrac{5 \times 7}{3 \times 7} = \dfrac{35}{21}, \dfrac{b}{c} = \dfrac{7 \times 3}{4 \times 3} = \dfrac{21}{12}$

    a : b : c = 35 : 21 : 12

11. $\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{5}{6}, \dfrac{c}{d} = \dfrac{7}{2}$

    $\dfrac{a}{b} = \dfrac{10}{15}, \dfrac{b}{c} = \dfrac{15}{18}$

    a : b : c = 10 : 15 : 18

    C is repeated twice

    $C : D = \dfrac{7 \times 18}{2 \times 18} = \dfrac{126}{36}$

    a : b : c = $10 \times 7 : 15 \times 7 : 18 \times 7$

    $= 70 : 105 : 126$

    a : b : c : d = 70 : 105 : 126 : 36

12. If $3A = 5B = 7C$, Find $A : B : C$

$\rightarrow$ $A : B : C = B \times C : C \times A : A \times B$

$A : B : C = 35 : 21 : 15$

13. If $A : B : C = \dfrac{7}{3} : \dfrac{5}{6} : \dfrac{9}{2}$

$\rightarrow$ $A : B : C = 7 \times 2 : 5 \times 1 : 9 \times 3$

$A : B : C = 4 : 6 : 27$

PROPORTION

The equality of two ratios is called proportion. a, b, c, d are said to be in proportion if $a : b = c : d$ or $a : b :: c : d$.

In a proportion, the first and fourth are known as extremes, while second and third terms are known as means. Hence a and d are extremes and b and c are means of the proportion $a : b :: c ; d$

In a proportion we always have:

Product of extremes=Product of means

$a \times d = b \times c$

CONTINUED PROPORTION

Four quantities a, b, c, d are said to be in a continued proportion, if

$a : b = b : c = c : d$ or $\dfrac{a}{b} = \dfrac{b}{c} = \dfrac{c}{d}$

Three quantities are said to be in continued proportion, if $a : b = b : c$

or $ac = b^2$

In this relationship, b is said to be the mean proportional between a and c and c is said to be a third proportional to a and b

Example 1 An object 1.6 m long casts a shadow 1.4 m long. At the same time another object kept nearbty casts a shadow 6.2 m long. Find the length of the second object.

Solution: Ratio of length of the object to its Shadow would be same.

$\therefore$ $1.6 : 1.4 = x : 6.2$

or

$x = \dfrac{1.6 \times 6.2}{1.4} = 7.08 m$

Some Results on Ratio and Proportion

i. Invertendo- If $a : b :: c:d$, then $b:a :: d:c$

ii. Alternendo- If $a : b :: c :d$, then $a:c :: b:d$

iii. Componendo- If $a : b :: c : d$, then

$(a+b) : b :: (c+d) : d$

iv. Dividendo - If $a : b :: c : d$, then

$(a - b) : b :: (c+d) : d$

v. Componendo and Dividendo -

If $a : b :: c : d$, then $(a + b) : (a - b) :: (c + d) : (c - d)$.

Equating the components of two and more ratios

If $\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{4}{5}$ Then find $a : b : c$.

To equate, common component b in the two ratios, take the LCM of 3 and 4 which is 12. Hence the new ratios are obtained as

$\dfrac{a}{b} = \dfrac{2 \times 4}{3 \times 4} = \dfrac{8}{12}, \dfrac{b}{c} = \dfrac{4 \times 3}{5 \times 3} = \dfrac{12}{15}$

Now, since the common component b in the two ratios has the equal values $a : b : c = 8 : 12 : 15$.

Therefore, if $\dfrac{a}{b} = \dfrac{n_1}{d_1}, \dfrac{b}{c} = \dfrac{n_2}{d_2}$

$\therefore$ $a : b : c = n_1 \times n_2 : d_1 \times n_2 : d_1 \times d_2$

Example 2 If $\dfrac{a}{b} = \dfrac{3}{5}, \dfrac{b}{c} = \dfrac{4}{7}$, find b : c

Solution: a : b : c = 3 × 4 : 5 × 4 : 5 × 7

= 12 : 20 : 35

Suppose there are three ratios

$$\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{4}{5} \text{ and } \dfrac{c}{d} = \dfrac{7}{15}$$

Now to find the value of a : b : c : d First of all we equate the common term b in the first two ratio and then the same process is repeated to equate the common term c.

LCM of 3 and 4 = 12

$$\therefore \quad \dfrac{a}{b} = \dfrac{2 \times 4}{3 \times 4} = \dfrac{8}{12} \text{ and } \dfrac{b}{c} = \dfrac{4 \times 3}{5 \times 3} = \dfrac{12}{15}$$

LCM of 15 and 7 = 105

$$\therefore \quad \dfrac{b}{c} = \dfrac{12 \times 7}{15 \times 7} = \dfrac{84}{105} \text{ and } \dfrac{c}{d} = \dfrac{7 \times 15}{15 \times 15} = \dfrac{105}{225}$$

Again we have to equate b for first two ratios

$$\therefore \quad \dfrac{a}{b} = \dfrac{8 \times 7}{12 \times 7} = \dfrac{56}{84}, \dfrac{b}{c} = \dfrac{84}{105}, \dfrac{c}{d} = \dfrac{105}{225}$$

Hence, a : b : c : d= 56 : 84 : 105 : 225

Therefore,

if $\dfrac{a}{b} = \dfrac{n_1}{d_1}, \dfrac{b}{c} = \dfrac{n_2}{d_2}$ and $\dfrac{c}{d} = \dfrac{n_3}{d_3}$

Then a : b : c : d = $n_1 \times n_2 \times n_3$:

$d_1 \times n_2 \times n_3 : d_2 \times n_3 : d_3 \times d_2 \times d_1$

Example 3

if $\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{4}{5}$ and $\dfrac{c}{d} = \dfrac{7}{11}$ find a : b : c : d

Soultion: a=2 × 4 × 7=56, b=3 × 4 × 7=84,

c=5 × 3 × 7=105

and   d=11 × 5 × 3=165

∴   a : b : c : d = 56 : 84 : 105 : 165

Example 4 if $\dfrac{a}{b} = \dfrac{1}{3}, \dfrac{b}{c} = \dfrac{4}{5}, \dfrac{c}{d} = \dfrac{7}{9}$ find a : b : c : d

Solution:

a=1 × 4 × 7=28, b=3 × 4 × 7=84,

c=5 × 3 × 7=105, d=3 × 5 × 9=135

∴   a : b : c : d = 28 : 84 : 105 : 135

Example 5 Three numbers are in the ratio 3 : 4 : 5, the sum of whose squares is 800. Find the numbers.

Solution Let the numbers be 3x, 4x and 5x

Then, $9x^2 + 16x^2 + 25x^2 = 800$

or, $50x^2 = 800 \Rightarrow x^2 = 16$ or x=4

So, the numbers are 12, 16 and 20.

Example 6 A and B are two alloys of gold copper prepared by mixing metals in proportions 7 : 2 and 7 : 11 respectively. If equal quantities of alloys are melted to form a third alloy C, then find the proportion of gold and copper in C.

Solution: In alloy C, gold = $\left(\dfrac{7}{9} + \dfrac{7}{18} \right) = \dfrac{21}{18}$

and copper

$$= \left(\dfrac{2}{9} + \dfrac{11}{18} \right) = \dfrac{15}{18}$$

∴   Ratio of gold and copper

$$= \dfrac{21}{18} : \dfrac{15}{18} = 7 : 5$$

Example 7 *A and B started a joint firm. A's investment was thrice the investment of B and period of his investment was two times the period of investment of B. If b got Rs. 4000 as profit, findtheir total profit.*

Solution: Ratio of investment of A and B = $3 \times 2 : 1 \times 1 = 6 : 1$

Share of B $= \dfrac{1}{7} \times$ Total profit $= 4000$

$\therefore$ Total Profit = Rs. 28000

Example 8 if $\dfrac{a}{b+c} = \dfrac{b}{c+a} = \dfrac{c}{a+b}$ *what is the value of each of the fractions? given that (a, b, c>0).*

Solution If each of the ratio is equal to k, then a=(b+c) K, b= (c+a) K and c=(a+b)K

Hence,   a+b+c=(2a+2b+2c) K

or   (a+b+c) - 2K (a+b+c)=0

or   (a+b+c) (1-2K)=0

So, $K = \dfrac{1}{2}$

Example 8 *If a : b=c : d and e : f = g : h find (ae+bf) : (ae-bf).*

Solution $\dfrac{a}{b} = \dfrac{c}{d}$ and $\dfrac{e}{f} = \dfrac{g}{h}$

$\therefore$ $\dfrac{ae}{bf} = \dfrac{cg}{dh}$

$\dfrac{ae + bf}{ae - bf} = \dfrac{cg + dh}{cg - dh}$

(Applying Componendo and Dividendo)

DIRECT PROPORTION

If A is directly proportional to B then as A increases B also increases proportionally. For example the relationship between speed, distance and time, speed is directly proportional to distance when time is constant. Hence, if speed is doubled distance travelled will also be doubled when time is kept constant. It is therefore important to note here that the variation is direct and proportional. If one quantity is doubled the related quantity will also be doubled.

Other examples of direct proportion are

(a) Simple interest Vs time (Principal and rate being constant)

(b) Density Vs mass (Volume being constant)

(c) Force Vs acceleration (mass being constant)

DIRECT VARIATION

If A is said to vary directly as B, then as A increases B also increases but not proportionally. This variation is denoted by A α B or A = KB, Where K is a constant.

For example, the total cost of productionis directly related to the number of items being produced. Here the variation is direct but not proportional.

INVERSE PROPORTION

A is inversely proportional to B means if A increases B decreases proportionally. If speed is doubled, time taken to cover the same distance is reduced to half.

Other examples of inverse proportion are

(a) Density Vs volume (mass being constant)

(b) Number of persons Vs time taken to complete the work. (work being same)

INVERSE VARIATION

If A inversely related to (or) varies inversely as B, then if B increases as A decreases but not proportionally. This relation can be expressed mathematically as

$A \alpha \dfrac{1}{B} \Rightarrow A = K \times \dfrac{1}{B}$ where K is a constant.

Here the variation is inverse but proportional.

Example 9 *A can do a piece of work in 12 days, B is 60% more efficient than A. Find the number of days that B takes to do the same piece of work.*

Solution

Ratio efficiencies of A and B

$= 100 : 160 = 5 : 8$

Since efficiency is inversely proportional to the number of days, hence ratio of days taken to complete the job is $8 : 5$

So, number of days taken by

$B = \dfrac{5}{8} \times 12 = 7\dfrac{1}{2}$ days.

Example 10 *The weight of the circular disc varies as the square of the radius when the thickness remains the same; it also varies as the thickness when the radius remains the same, Two discs have their thickness in the ratio 9 : 8. Find the ratio of the radii if the weight of the first is twice that of the second.*

Solution: $\qquad W \, \alpha \, T \times (r)^2$

$\therefore \qquad W_1 = 9 \times (r_1)^2$

and $\qquad W_2 = 8 \times (r_2)^2$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{W_1}{W_2} \times \frac{8}{9} = \frac{2W_2}{W_2} \times \frac{8}{9} = \frac{16}{9}$$

$\qquad\qquad\qquad\qquad [W_1 = 2W_2, \text{ given})$

$\therefore \qquad r_1 : r_2 = 4 : 3$
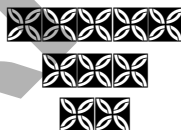
1. If $a : b = c : d$, then the value of $\dfrac{a^2 + b^2}{c^2 + d^2}$ is

   (a) $\dfrac{1}{2}$ (b) $\dfrac{a+b}{c+d}$

   (c) $\dfrac{a+b}{c-d}$ (d) $\dfrac{ab}{cd}$

2. If $A : B = 3 : 4$, $B : C = 8 : 9$, $C : D = 15 : 16$ Find $A : B : C : D$

   (a) $15 : 20 : 21 : 28$

   (b) $30 : 40 : 45 : 48$

   (c) $9 : 15 : 21 : 28$

   (d) none of these

3. What should be subtracted from each of the numbers 54, 71, 75 and 99 so that the remainders are in continued proportion?

   (a) 9   (b) 7

   (c) 4   (d) 3

4. Gold is 19 time as heavy as water and copper 9 times as heavy as water. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?

   (a) $2 : 5$ (b) $2 : 3$

   (c) $3 : 1$ (d) $3 : 2$

5. A mixture of 729 mL contains milk and water in the ratio 7 : 2. How much more water is added to get a new mixture containing milk and water in the ratio 7 : 3?

   (a) 81 mL   (b) 72 mL

   (c) 80 mL   (d) 78 mL

6. Ratio of boys to the girls in a class is 5 : 4. Which of the following cannot be the number of students in the class?

   (a) 45   (b) 72

   (c) 108   (d) 98

7. The ratio of spirit and water in a mixture is 1 : 3. If the volume of the solution is increased by 25% by adding spirit only. What is the resultant ratio of spirit and water?

   (a) $2 : 3$ (b) $1 : 4$

   (c) $1 : 2$ (d) none of these

8. An amount of money is to be distributed among A, B and C is the ratio 5 : 8 : 12 respectively. If the total share of B and C is four time that of A. What is A's share?

   (a) Rs. 3000   (b) Rs. 5000

   (c) cannot be dtermined

   (d) None of these

9. If there are Rs. 495 in a bag in denominations of one rupee, 50 paisa and 25 paisa coins. Which are in the ratio 1 : 8 : 16. How many 50 paisa coins are there in the bag?

   (a) 50   (b) 220

   (c) 440   (d) none of these

10. Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio be comes 5 : 7. The numbers are

    (a) 3, 5   (b) 7, 9

    (c) 13, 22   (d) 15, 25

11. One year ago the ratio between Laxman's and Gopal's salary was 3:4. The ratio's of their individual salaries between last year's and this year's salaries are 4 : 5 and 2 : 3 respectively. At present the total of their salary is Rs. 4160. The salary of

Laxman, now is

(a) Rs. 1040   (b) Rs. 1600

(c) Rs. 2560   (d) Rs. 3120

12. If x varies inversely as y^2+1 and equal to 6 when y=8. Find x when y=7

(a) 7.8   (b) 11.3

(c) 12   (d) 9

13. A disnonest milkman mixed of 2 L of water for every 5 L of milk and thus prepared a mixture of milk and water if 49 L. If he finally adds 14 L of pure milk to the mixture, find the ratio of milk and water in the mixture.

(a) 12 : 5   (b) 14 : 3

(c) 7 : 21   (d) 9 : 4

14. A vessel contains liquids A and B in the ratio 3 : 1. If 8 L of mixture are removed and the same quantity of liquid B is added, the ratio becomes 1:3. What quantity does the vessel hold?

(a) 12L   (b) 14L

(c) 16L   (d) 10L

15. The ratio of two numbers is 2 : 3. If each number is decreased by 3 the ratio becomes 3 : 5. Find the difference of numbers.

(a) 5   (b) 7

(c) 6   (d) 4

16. The incomes of A and B are in the ratio 4 : 3 and their expenditure are in the ratio 5 : 2. If each seves Rs. 4900, find the expenses of B.

(a) Rs. 1400   (b) Rs. 1600

(c) Rs. 1800   (d) Rs. 2000

17. A mixture contains milk and water in the ratio of 6 : 1. On adding 4L of water, the ratio of milk to water becomes 6 : 5. Find the quantity of water in the mixture.

(a) 6 L   (b) 56L

(c) 60 L   (d) 2L

18. The sum of three numbers is 275. If the ratio between the first and second be 3 : 7 and that between the second and third be 2 : 5 then find the second number.

(a) 30   (b) 75

(c) 70   (d) 80

19. A and B invest Rs. 3000 and 4000 in a business. A receives Rs. 10 per month out of the profit as a remuneration for running the business and the rest of profit is divided in proportion to the investments. If in a year 'A' totally receives Rs. 390. What does B receive?

(a) Rs. 630   (b) Rs. 360

(c) Rs. 480   (d) Rs. 380

20. Rs. 535 is divided among A, B and C so that if 15, Rs. 10, Rs. 30 be subtracted from their respective shares, the remainders may be in the ratio 4 : 5 : 7. What was their initial shares (in Rs.)?

(a) 135, 160, 240   (b) 160, 280, 310

(c) 150, 210, 325   (d) None of these

21. A driver's income consists of his salary and tips. During one week his tips were $\frac{5}{4}$ of his salary. What fraction of his income came from tips?

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$

(c) $\frac{5}{8}$ (d) $\frac{5}{4}$

22. Several litres of acid were drawn off a 54L vessel full of acid and an equal amount of water added. Again the same volume of the mixture was drwan off and replaced by water. As a result the vessel contained 24L of pure acid. How much of the acid was drawn off initially?

(a) 12L   (b) 16L

(c) 18L   (d) 24L

23. Rs. 6500 were divided equally among a cretain number of persons. Had there been 15 more persons each would have got Rs. 30 less. Find the original number of person.

(a) 50   (b) 60

(c) 40   (d) 55

24. What is the value of $\frac{P+Q}{P-Q}$ if $\frac{P}{Q}=7$?

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$

(c) $\frac{2}{6}$ (d) $\frac{7}{8}$

25. Rs. 770 have been divided among A, B and C such that A receives $\frac{2}{9}$ th of what B and C together receive. Then A's share is

(a) Rs. 140   (b) Rs. 154

(c) Rs. 165   (d) Rs. 170

26. A sum of Rs. 370 is to be divided among A, B and C such that $\frac{A's\ share}{B'\ share}=\frac{B's\ share}{C's\ share}=\frac{3}{4}$. Then A'share is.

(a) Rs. 240   (b) Rs. 120

(c) Rs. 100   (d) Rs. 90

27. In a mixture of 45 L the ratio of milk and water is 3 : 2. How much water must be added to make the ratio 9:1?

(a) 10L   (b) 15L

(c) 17L   (d) 20L

28. Three containers A, B and C, are having mixtures of milk and water in the ratio 1 : 5, 3 : 5 and 5 : 7 respectively. If the capacities of the containers are in the ratio 5 : 4 : 5, then find the ratio of the milk to the water if the water if the mixtures of all the three containers are mixed together.

(a) 51 : 115   (b) 52 : 115

(c) 53 : 115   (d) 54 : 115

29. Profits of a business are distributed among three parters A, B and C in cuch a way that 4 times that amount reveice by A is equal to 6 times the amount received by B and 11 times the amount received by C. The ratio in which the three received the amount is:

(a) 4 : 6 : 11   (b) 11 : 6 : 4

(c) $\frac{1}{4}:\frac{1}{6}:\frac{1}{11}$ (d) 66 : 44 : 24

30. A part of monthly expenses of a family is constant and the remaining varies with the price of wheat. When the rate of wheat is Rs. 250 a quintal, the total monthly expenses of the family are Rs. 1000 and when it is Rs. 240 a quintal, the total monthly expenses are Rs. 980. Find the total monthly expenses of the family when the cost of wheat is Rs. 250 a quintal.

(a) Rs. 1000   (b) Rs. 1400

(c) Rs. 1200   (d) Rs. 800

31. An iron cube of size 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet be in the ratio 1:5, then the sides are

(a) 20 cm, 100 cm (b)   10 cm, 50 cm

(c) 40 cm, 200 cm (d)   none of these

32. If three equal cubes are placed adjacently in a row, then the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of the three cubes will be.

(a) 5 : 9   (b)   1 : 3

(c) 2 : 3   (d)   7 : 9

33. Two vessels contain mixtures of milk and water in the ration 8:1 and 1 : 5 respectively. The contents of both of these are mixed in a specific ratio into a third vessel. How much mixture must be drawn from the second vessel to fill the third vessel (capacity 26 gallons) completely in order that the resulting mixture may be half milk and half water?

(a) 12 gallons   (b)   14 gallons

(c) 10 gallons   (d)   13 gallons

34. Fresh grapes contain 80% water while dry grapes contain 10% water. If the weight of dry grapes is 250 kg.What was its total weight when it was fresh?

(a) 1000 kg   (b)   1125 kg

(c) 1225 kg   (d)   1100 kg

35. A misture (40L) contains conic and water in the ratio 3 : 1. To make the ratio 5 : 2 how much additional amount of water is required?

(a) 5 L   (b)   1 L

(c) 3 L   (d)   2 L
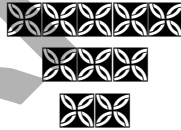
1. **(d)** $\dfrac{a}{b}=\dfrac{c}{d}$, Hence, $\dfrac{a^2+b^2}{c^2+d^2}=\dfrac{b^2\times\dfrac{c^2}{d^2}+b^2}{c^2+d^2}$

$$=\dfrac{b^2\left(c^2+d^2\right)}{d^2\left(c^2+d^2\right)}=\dfrac{b^2}{d^2}$$

Now, $\dfrac{b^2}{d^2}=\dfrac{b\times b}{d\times d}=\dfrac{a}{c}\times\dfrac{b}{d}=\dfrac{ab}{cd}$

2. **(b)** $A:B:C:D=n_1\times n_2\times n_3:$

$d_1\times n_2\times n_3:d_2\times d_1\times n_3:d_3\times d_2\times d_1$

$=3\times 8\times 15:4\times 15:4\times 9\times 16$

$=30:40:45:48$.

3. **(d)** Let number x be subtracted from each of the numbers, then

$\dfrac{54-x}{71-x}=\dfrac{75-x}{99-x}$. For x=3, this

relation is correct.

4. **(d)** Let 1 gm of gold be mixed with x gm of copper,

then 1 gm of gold + x gm of copper = (1+x) gm mixture.

$\Rightarrow\qquad x=\dfrac{2}{3}$

$\therefore\qquad$ Ratio $==1:\dfrac{2}{3}=3:2$

5. **(a)** The volume of new solution is $\dfrac{10}{9}$ times the original volume. Hence if original volume is 729 ml, the new voulue=810 ml. The increase is due to the addition of (810-729) = 81 ml. of water.

6. **(d)** The total number of students should be divisible by (5+4)=9.

7. **(a)** Let the volume of spirit and water be x and 3x. Then total volume = 4x. Resultant volume of solution.

$=1.25\times 4x=5x$

Therefore, increase in volume.

$=5x-4x=x$

So, the new ratio of spirit to water.

$2x:3x=2:3$

It is to be noted that increase in volume is due to additional of spirit only.

8. **(c)** It cannot be determined because the total money to be distributed is not given.

9. **(c)** Ratio of number of coins =1:8:16

Ratio of value of each type of coins

$=1:4:4$.

$\therefore$ Value of 50 paisa coins

$=\dfrac{4}{9}\times 495=220$

Therefore, number of coins of 50 paisa = 200 $\times 2$=440.

10. **(d)** $\dfrac{3x+10}{5x+10}=\dfrac{5}{7}\Rightarrow 21x+70=25x+50$

$\Rightarrow\qquad x=5$

$\therefore\quad$ Number are 15 and 25

11. **(b)** Let the salaries of Laxman and Gopal one year before be L_1, G_1 respectively and now L_2, G_2 respectively. Then,

$$\frac{L_1}{G_1} = \frac{3}{4}, \frac{L_1}{L_2} = \frac{4}{5}, \frac{G_1}{G_2} = \frac{2}{3}$$

and $\qquad L_2, G_2 = 4160$.

Solving these equations, we get L_2 = Rs. 1600

12. **(a)** $x \propto \dfrac{1}{y^2+1} \Rightarrow x = \dfrac{K}{y^2+1}$

or, $\quad 6 = \dfrac{K}{8^2+1} \Rightarrow K = 6 \times 65 = 390$

$\therefore \quad x = \dfrac{390}{7^2+1} = \dfrac{390}{50} = 7.8$

13. **(c)** Quantity of milk and water in 49L of mixture = 35L and 14L respectively. After adding 14L of pure milk, quantity of milk in the mixture = 49L. Therefore ratio of milk and water = 7 : 2.

14. **(a)** Let the quantity of liquids A and B in the mixture be 3x and x. Quantity of liquids A and B after removal of 8L of mixture= (3x-6) and (x-2). So, ratio of A and B after adding 8L of

$\qquad$ B=(3x-6) : (x+6)=1 : 3

$\Rightarrow \qquad$ x = 3

15. **(c)** $\dfrac{2x-3}{3x-3} = \dfrac{3}{5} \Rightarrow 10x - 15 = 9x - 9 \quad$ or

x=6. Therefore,

difference of number

$= 3 \times 6 - 2 \times 6 = 6$

16. **(a)** Let the income of A and B be Rs. 4x and Rs. 3 x respectively and their expenses be Rs. 5y and 2y respectively, then their saving 4x-

5y=4900 and 3x-2y = 4900 and 3x-2y = 4900. Solving these equations we get y=700. Hence, expenses of B=700×2=Rs. 1400

17. **(c)** $\dfrac{6x}{x+4} = \dfrac{6}{5} \Rightarrow 30x = 6x + 24 \text{ or } x = 1$.

So, quantity of water in the mixture = 1 L.

18. **(c)** $\dfrac{A}{B} = \dfrac{3}{7}, \dfrac{B}{C} = \dfrac{2}{5} \Rightarrow A:B:C = 6:14:35$

$\therefore$ Second number$= \dfrac{14}{55} \times 275 = 70$

19. **(b)** Ratio of investments of A and B =3 : 4

Amount of profit received by A=(390-120) = Rs. 270

$\therefore \quad \dfrac{3}{7} \times x = 270 \Rightarrow x = Rs\, 630$

$(x = total\, profit)$

So, Share of B=$\dfrac{4}{7} \times 630$=Rs. 360

20. **(a)** Total amount deducted = Rs. 55
Amount left = 535-55=Rs. 480

$\therefore \qquad$ A's share = Rs. 120, B's share $\quad$ = Rs. 150 and C's share $\qquad$ = Rs. 210

So, initial share of A, B and C
$\qquad$ = Rs. 135
Rs. 160 and Rs. 240

21. **(b)** Let his basic salary be x,

Then, his income $= x + \dfrac{5}{4}x = \dfrac{9x}{4}$

$\therefore$ Reqwuired fraction

$$= \frac{\frac{5}{4}x}{\frac{9}{4}x} = \frac{5}{9}$$

22. (c) **Note:** *If a contrainer contains x units of liquid and y units of liquid is taken out. If this operation is repeated n time. The final quantity of the liquid in the container is.*

$$x\left(1 - \frac{y}{x}\right)^n$$

Hence, in this question

$$24 = 54\left(1 - \frac{x}{54}\right)^2$$

where x = amount of acid initially drawn off. $\left(1 - \frac{x}{54}\right)^2 = \frac{24}{54} = \frac{4}{9}$

$\Rightarrow \quad \left(1 - \frac{x}{54}\right) = \frac{2}{3} \Rightarrow \frac{x}{54} = \frac{1}{3}$

or $\qquad$ x=18L

23. (a) Let there be originally x persons. Then share of each person

$$= Rs \frac{6500}{x}$$

Given, $\quad \frac{6500}{(x+15)} = \frac{6500}{x} - 30$

$\Rightarrow \quad \frac{6500}{x} - \frac{6500}{(x+15)} = 30$

Using options we final that x = 50

24. (a) $\dfrac{P+Q}{P-Q} = \dfrac{\dfrac{P}{Q}+1}{\dfrac{P}{Q}-1} = \dfrac{7+1}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}$

25. (a) A+B+C=770

$$A = \frac{2}{9}(B+C)$$

From Eqs. (i) and (ii), we have

$$A = \frac{9}{2}A = 770$$

$\Rightarrow \qquad 11A = 770 \times 2$

$\Rightarrow \qquad A = 140$

26. (d) $\dfrac{A}{B} = \dfrac{3\times3}{4\times3} = \dfrac{9}{12} : \dfrac{B}{C} = \dfrac{3\times4}{4\times4} = \dfrac{12}{16}$

$\therefore \qquad A : B : C = 9 : 12 : 16$

$\therefore$ As share $= \dfrac{9}{37} \times 370 = Rs.90$

27. (b) Quantity of milk $= \dfrac{3}{5} \times 45 = 27L$

Quantity of water $= \dfrac{2}{5} \times 45 = 18L$

Let x litre of water be added to get the ratio 9 : 11

Then, $\qquad \dfrac{18+x}{27} = \dfrac{11}{9}$

$\Rightarrow \qquad 162 + 9x = 297$

$\Rightarrow \qquad 9x = 135 \text{ or } x = 15$

28. (c) Ratio of milk and water

$$= \left(\frac{1}{6} \times 5 + \frac{3}{8} \times 4 + \frac{5}{12} \times 5\right):$$

$$\left(\frac{5}{6} \times 5 + \frac{5}{8} \times 4 + \frac{7}{12} \times 5\right)$$

$= \qquad 106 : 230$

$= \qquad 53 : 115$

29. (c)

30. (c) Let constant expenses of the family = E and they buy x amount of wheat

Then, $E+x \times 250 = 1,000$

$E+x \times 240 = 980$

$\therefore \qquad\qquad x = 2$

$E = 500$

31. (a) Equating the volume : $x \times 5x \times 0.5 = 13^3$. On solving we get $x = 20$. So the sides are 20 and 100.

32. (d) We get the following diagram

Surface area of the entire cubiod = 14

Surface area of individual cubes = $6 \times 3 = 18$

Ratio = 14 : 18 or 7 : 9

33. (b) Let us take 18 in each (LCM of 6 and 9)

Then milk ratio in the two containers is 16 : 3. In the third vessel the ration is 9 : 90. So, applying alligation, we get the ration 6 : 7.

Required quantity $\frac{7}{13 \times 26} = 14$

34. (b) In 250 kg there is 225 kg pulp and 25kg liquid.

Now 225=20% (Because 80% water is contained in grapes)

$\therefore$ Total weight of fresh grapes $= 225 \times 5 = 1125$

35. (d) In 40 conic = 30 and water=10

Now $\frac{30}{10+x} = \frac{5}{2}$

$\Rightarrow \qquad\qquad 60 = 50 + 5x$

$\Rightarrow \qquad\qquad x = 2L$

9 Mixture and Alligations
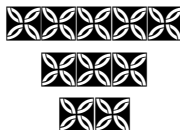
This chapter is an extension of the previous chapter ratio and proportion. In fact this chapter deals with a specific, type of questions which can be solved quickly using the method of Alligations. Hence it is very necessary to understand and identify the pattern of such questions so as to apply the rules of alligations.

Alligation is the rule to find the proportion in which the two or more quantities at the given price must be mixed to produce a mixture at a given price.

Rule of Alligation : If two quantities are mixed in a ratio, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{CP of dearer} - \text{Mean price}}{\text{Mean Price} - \text{CP of cheaper}}$$

The above relationship can also be represented as under

CP of a unit quantity of cheaper (c)   CP of unit Quantity of dearer (d)

Example 1 *Two varieties of tea costing Rs. 12 per kg and Rs. 15 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.*

Solution: Using the above rule we get.

$$\frac{1}{2} = \frac{15-m}{m-12} \text{ or,} (m-12) = (30-2m)$$

or $3m = 42 \Rightarrow m = $ Rs. 14 per kg

An understanding of the above rule can enable a student to use it in variety of ways.

Example2 On combining two groups of students having 30 and 40 marks respectively in an exams, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution: $\dfrac{x_1}{x_2} = \dfrac{40-m}{m-20} = \dfrac{40-34}{34-30} = \dfrac{6}{4}$

$\therefore$ $x_1 : x_2 = 6 : 4$

Example 3 *In what proportion must rice at Rs. 3.10 per kg. be mixed with rice at Rs. 3.60 per kg, so that the mixture be worth Rs. 3.25 a kg?*

Solution: $\dfrac{x_1}{x_2} = \dfrac{3.60-3.25}{3.25-3.10} = \dfrac{35}{15} = \dfrac{7}{3}$

Hence,   Required ratio = 7 : 3

Example 4 *In what ratio must three kinds of rice Rs. 12, Rs. 14 and Rs. 16 per kg be mixed so as to obtain a mixture worth Rs. 13 kg?*

Solution: In such questions involving three quantities, find the ratio of first and second to form the desired mixture, then find the ratio of first and third to the desired mixture.

$$\frac{\text{Ist}}{\text{IInd}} = \frac{14-13}{13-12} = \frac{2}{1}$$

$$\frac{\text{Ist}}{\text{IIIrd}} = \frac{16-13}{13-12} = \frac{3}{1}$$

$$\therefore \quad \frac{\text{IInd}}{\text{IIIrd}} = \frac{\text{IInd}}{\text{Ist}} \times \frac{1\text{st}}{\text{IIIrd}} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$$

Now $\dfrac{\text{Ist}}{\text{IInd}} = \dfrac{2}{1}, \dfrac{\text{IInd}}{\text{IIIrd}} = \dfrac{3}{2}$

Therefor, Ist : IInd : IIInd = 6 : 3 : 2

Example 4 *A butler stole wine from a butt of cherry, which contained 40% of spirit solution. He replaced what he had stolen with wine containing only 16% spirit by volume. The resultant concentration of the mixture in the butt was then 24%. What fraction of the butt of cherry did he steal?*

Solution:

Wine containing   Wine containing

40% spirit   16% spirit

Wine containing

24% spirit

8   16
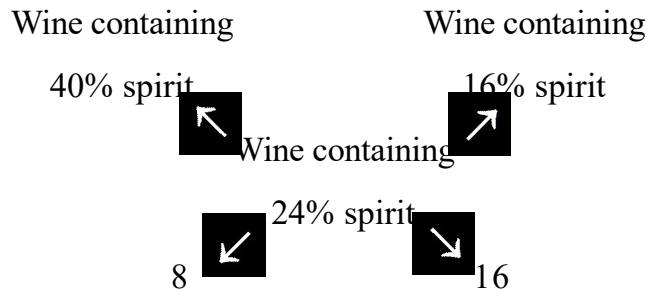
$$\frac{\text{wine with 40\% spirit}}{\text{wine with 16\% spirit}} = \frac{8}{16} = \frac{1}{2}$$

They must be mixed in the ratio 1 : 2.

Hence, $\frac{1}{3}$ of the butt of cherry was left and

hence the butler drew out $\frac{2}{3}$ of the butt.

Note:- If a container originally contains x units of liquid and y units of liquid is taken out. If this operation is repeated n times.

Then final quantity of the liquid in the

container is $x\left(1 - \dfrac{y}{x}\right)^n$ units.

Example 5 *From a container which has 120 L of milk, 40 L of milk is taken out and is replaced with water. This process is repeated twice. How much milk is in the container now?*

Solution: Quantity of milk in the container.

$$= 120\left(1 - \frac{40}{120}\right)^2$$

$$= 120\left(1 - \frac{1}{3}\right)^2 = 120 \times \frac{4}{9} = 53.33 \text{L}$$

Example 6 *A dishonest milkman professes to sell pure milk at cut price, but he adds water in it and thereby gains 25%. Find the percentage of water in the mixture.*

Solution: *Let CP of milk be Re 1 per litre.*

Then SP of the mixture = Rs. 1 per litre and

CP of mixture $= \dfrac{1}{1.25} = \text{Re} \dfrac{4}{5}$

$$\frac{x_1}{x_2} = \frac{1 - \dfrac{4}{5}}{\dfrac{4}{5} - 0} = \frac{1}{5} \times \frac{5}{4} = \frac{1}{4}$$

(where x_1 = quantity of water and x_2 = quantity of milk).

Hence percentage of water in the mixture

$$= \frac{1}{5} \times 100 = 20\%$$
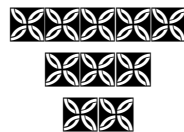
1. Gold is 19 time as heavy as water and copper 9 times as heavy as water. In what ratio should these metal be mixed so that the mixture may be 15 times as heavy as water?

    (a) 3 : 2   (b) 9 : 5

    (c) 2 : 3   (d) 7 : 5

2. Two liquids are mixed in the ratio 3: 5 and the mixture is sold at Rs. 120 with a prifit of 20%. If the first liquid is costlier than the second by Rs. 2 per litre, find the cost of the costlier liquid per litre.

    (a) Rs. 92.30   (b) Rs. 74.10

    (c) Rs.101.25   (d) Rs. 99.25

3. Two vessela A and B contain milk and water mixed in the ratio 5 : 2 and 8 : 5 respectively. Find the ratio in which these mixtures are to be mixed to get a new mixture containing milk and water in the ratio 9 : 4.

    (a) 5 : 2   (b) 9 : 7

    (c) 5 : 3   (d) 7 : 2

4. A grocer buys two kinds of rice at Rs. 1.80 and Rs. 1.20 per kg respectively. In what proportion should these be mixed, so that by selling the mixture at Rs. 1.75 per kg, 25% may be gained

    (a) 20 kg   (b) 30 kg

    (c) 15 kg   (d) 35 kg

5. Sohanlal mixes 80 kg of sugar worth Rs. 6.75 per kg with 120 kg worth Rs. 8 per kg. At what rate shall he sell the mixture to gain 20%?

    (a) Rs. 7.50   (b) Rs. 9

    (c) Rs. 8.20   (d) Rs. 8.85

6. A can contains a mixture of two liquids A and B in proportion 7 : 5. When 9 L of mixture are drawn off and the can is filled with B, the proportion of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?

    (a) 25 L   (b) 108 L

    (c) 45 L   (d) 63 L

7. A vessel contains milk and water in the ratio 3 : 2. The volume of the contents is increased by 50% by adding water to it. From this resultant solution 30L is withdrawn and then replaced with water. The resultant ratio of milk to water in the final solution is 3 : 7. Find the original volume of the solution.

    (a) 80 L   (b) 65 L

    (c) 75 L   (d) 82 L

8. 3 L water is taken out from a vessel full of water and substituted by pure milk. This process is repeated two more times. Finally the ratio of milk and water in the solution becomes 1728 : 27. Find the volume of the original solution.

    (a) 3 L   (b) 2 L

    (c) 4 L   (d) 9 L

9. A vessel contains 50 L milk.The milkman delivers 10 L to the first house and adds an equal qantity of water. He does exactly the same at the second and third house. What is the ratio of milk and water when he has finished delivering at the third house?

(a)   61 : 64   (b)   27 : 37

(c)   16 : 19   (d)   None of these

10. Several litres of acid drawn off from a 54L vessel full of acid and an equal amount of water is added. Again the same volume of the mixture was drawn off and replaced by water. As a result, the vessel contained 24 L pure acid. How much acid wad drawn off initially?

(a)   12L   (b)   16L

(c)   18L   (d)   24L

11. A mixture of a certain quantity of milk 32 L of water is worth Rs. 1.50 per litre. If pure milk be worth Rs. 4.50 per litre how much milk is there is the mixture?

(a)   18L   (b)   14L

(c)   16L   (d)   20L

12. A person has a chemical of Rs. 50 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs. 40 per litre he may get a profit of 50%.

(a)   8 : 7   (b)   9 : 8

(c)   10 : 7   (d)   4 : 3

13. A trader has 50 kg. of rice, a part of which he sells at 14% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% proft?

(a)   30 kg   (b)   35 kg

(c)   40 kg   (d)   none of these

14. A container contains 7 part milk and 3 part water. How many parts of mixture should be taken out and replaced by water so that container contains half milk and half water?

(a) $\frac{3}{7}$ part   (b) $\frac{4}{7}$ part

(c) $\frac{2}{7}$ part   (d) $\frac{1}{7}$ part

15. A solution of sugar syrup has 15% sugar. Another solution has 5% sugar. How many litres of the second solution must be added to 20 L of the first solution to male a solution of 10% sugar?

(a)   10   (b)   5

(c)   15   (d)   20

16. In two alloys, copper and zinc are related in the ratios of 4 : 1 and 1 : 3 kg of 1st alloy, 16 kg of 2nd alloy and some of pure copper are melted together. An alloy was obtained in which the ratio of copper to zinc was 3 : 2. Find the weight of the new alloy.

(a)   34kg   (b)   35kg

(c)   30kg   (d)   27kg

17. Two solution of 90% and 97% purity are mixed resulting in 21L of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?

(a)   15L   (b)   12L

(c)   9L   (d)   6L

18. A gain percent of 20 is made by selling the mixture of two types of ghee at Rs. 230 ker kg. If the type costing Rs. 250 per kg is mixed with 225 kg of the other, how many kilograms of the former was mixed?

(a)   115 kg   (b)   85 kg

(c)   69 kg

(d)   cannot be determined

19. Ram went to shop to buy 50 kg of rice. He buys two kinds of rice which cost him Rs. 4.50 per kg and Rs. 5.00 per kg. He spend a total of Rs. 240. What was the quantity of rice bought which cost him Rs. 4.50 per kg?

(a) 25 kg   (b) 30kg

(c) 20 kg   (d) none of these

20. Two vessel contain mixtures of milk and water in the ratio of 8 : 1 and 1 : 5 respectively. The contents of both of these are mixed in a specific ratio into a third vessel.

How much mixture must be drawn from the second vessel to fill the third vessel (cepacity 26 gallons) completely in order that the resulting mixture may be half milk and half water?

(a) 12 gallons   (b) 14 gallons

(c) 10 gallons   (d) 13 gallons
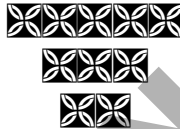
1. (a) Using the rule of alligation we find the ratio of gold and copper=3:2 as calculated below.

Gold 19   Copper 9

15

(15 – 9) 6   (19 – 15) 4
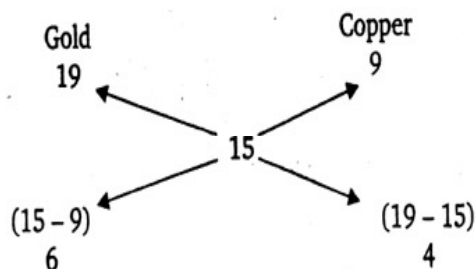
2. (c) CP of the mixture = Rs. 100

$$\therefore \quad \frac{100 - x}{(x + 2) - 100} = \frac{3}{5}$$

$$\Rightarrow \quad x = 99.25$$

3. (d) Let the CP of milk be Rs. 1 per litre.

$$\left(Rs \frac{5}{7}\right) \quad \text{mean price} \quad \left(Rs \frac{8}{13}\right)$$

$$\left(Rs \frac{9}{13}\right)$$

$$\frac{1}{13} \qquad \frac{2}{91}$$
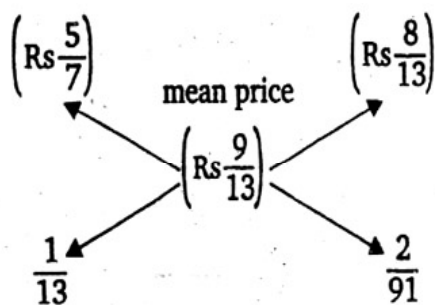
$$\therefore \text{(Mix in A)} : \text{(Mix in B)} = \frac{1}{13} : \frac{2}{91}$$

$$= 7 : 2$$

4. (b) Profit (I)   Profit (II)

8%   18%

Mean Profit (14%)

4   6
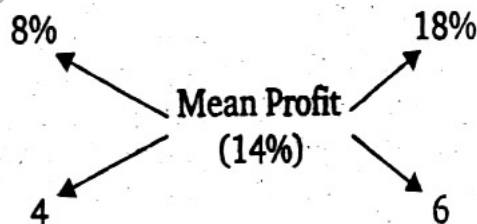
Ratio of Ist and 2nd part = 4 : 6 = 2:3

Now $\frac{x}{(50 - x)} = \frac{2}{3} \Rightarrow 3x = 100 - 2x$

or   x = 20 kg

$\therefore$ Quantity gold at 18% profit=50-x

= 50 - 20 = 30kg

5. (b) Total cost price of 200 kg of mixture

=Rs.($80 \times 6.75 + 120 \times 8$)= 1500

Average rate = Rs. 7.50 per kg

$\therefore$ Required selling price

$$= 7.50 \times 1.2$$

$$= \text{Rs. 9 per kg.}$$

6. (d) When 9 L is taken out and 9 L of B is added then.

$$\frac{\frac{7}{4}x}{\frac{5}{4}x + 9} = \frac{7}{9} \Rightarrow x = 9$$

7. (a) Let the original volume be x. Then quantity of milk and water

$$\frac{3x}{5} \text{ and } \frac{9x}{10} \text{ respectively.}$$

Now $\quad \dfrac{\dfrac{3x}{5} - 12}{\dfrac{9x}{10} + 12} = \dfrac{3}{7}$

$$\Rightarrow \quad 14 (3x-60) = 3(9x+120)$$

$$\Rightarrow \quad x = 80L$$

8. (c) Final quantity of water $= x\left(1 - \dfrac{y}{x}\right)^{n}$

$$= x\left(1 - \frac{3}{x}\right)^{3}$$

$$\frac{F}{x} = \left(\frac{x - 3}{x}\right)^{3} \Rightarrow \frac{27}{1728} = \left(\frac{x - 3}{x}\right)^{3}$$

or $\dfrac{x-3}{x} = \dfrac{3}{12}$ or x = 4L

9. (a) Final quantity of milk.

$$= 50\left(1 - \dfrac{10}{50}\right)^3 = 50 \times \dfrac{4}{5} \times \dfrac{4}{5} \times \dfrac{4}{5} = 25.6L$$

Hence quantity of water left in the solution = 40 - 25.6 = 24.4L
Hence ratio of milk and water
= 24.4 : 25.6 = 61.64

10. (c) $24 = 54\left(1 - \dfrac{x}{54}\right)^2 \Rightarrow \dfrac{24}{54} = \left(1 - \dfrac{x}{54}\right)^2$

or $\dfrac{4}{9} = \left(1 - \dfrac{x}{54}\right)^2 \Rightarrow x = 18L$

11. (c) Milk   Water


```
4.50            0
     1.50
 1.5            3
```

12. (a) Chemical   Water


```
50              0
     80
     ──
      3
 80            70
 ──            ──
  3             3
```

13. (a) I Part   II Part


```
 14          (-6)
      (-4)
-4-(-6)     14-(-4)
 =2           =18
```

14. (c) Mixture   Water


```
  3            1
 ──
 10
        1
        ─
        2
  1           1
  ─           ─
  2           5
  5     :     2
```
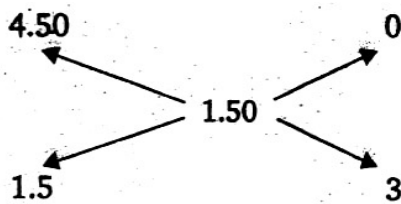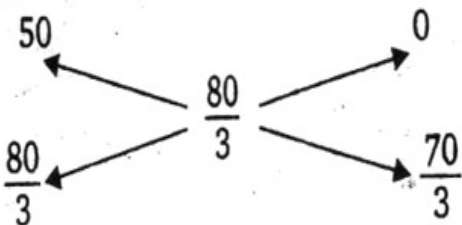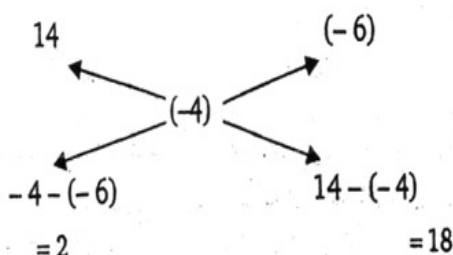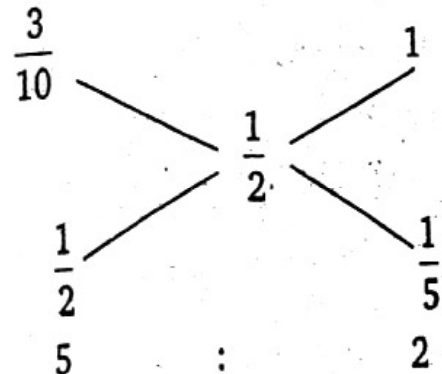
It is given that quantity of mixture taken out is equal in quantity of the water replaced $= \dfrac{2}{7}$ part.

15. (d) Ist type   IInd type


```
 15            5
      10
  5     :     5
```
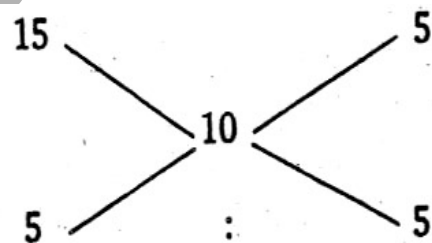
Hence both the types should be added in the ratio of 1 : 1 to obtain the required strength. Hence 20L of first type should be added to the 20L of the second type to get the desired solution.

16. (b) Let the quantity of the pure copper be x kg.

then, $x + \dfrac{4}{5} \times 10 + \dfrac{1}{4} \times 16$

$= \dfrac{3}{5}(10 + 16 + x)$

17. (c) Ist   IInd

$$\begin{matrix} 90 & & & 97 \\ & & 94 & \\ 3 & & & 4 \end{matrix}$$
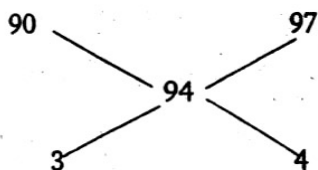
18. (d) To know the quantity of other one quantity either the quantity of first or quantity of second should be given.

Rate of Interest $= \dfrac{SI \times 100}{P \times t}$

$= \dfrac{882 \times 100 \times 2}{8400 \times 3} = 7\%$

19. (c) Let the quantity of nice at Rs. 4.50 per kg be x and that of Rs. 5 per kg. be (x-50).

Then, we have

$$4.5x + 5(50-x) = 240$$
$$250 - 5x + 4.5x = 240$$
$$0.5x = 10 = 20kg$$

20. (b)
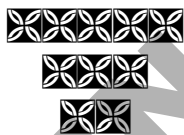
PROPORTIONALITY THEOREM

Intercepts made by two transversal lines (cutting lines) on three or more parallel lines are proportional. In the figure, lines Y_1, X_1 and $X_2 Y_2$ are transversal to three parallel lines AB, CD and BF. Then AC, CE, BD, DF are intercepts.
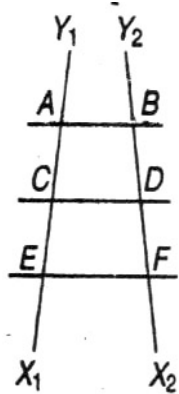
Applying proportionality theorem,

we find $\dfrac{AC}{BD} = \dfrac{CE}{DF}$

Properties of Triangles

1. Sum of three interior angles $= 180^0$

2. Exterior angle = Sum of two interior opposite angles.

3. Sum of any two sides is always greater than the third side.

4. Side opposite to the greatest angle will be greatest in length and vice-versa.

5. Let a, b and c be the sides of a triangle and c the largest side. Then,

   (i) if $c^2 < a^2 + b^2$, the triangle is acute angle.

   (ii) $c^2 = a^2 \, b^2$, the triangle is right angle.

   (iii) if $c^2 > a^2 + b^2$, the triangle is obtuse angle.

Sine Rule

If a, b and c are the sides opposite to angles A, B and C a b respectively, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
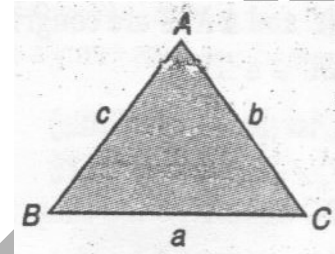
These rules are helpful in finding the third side if we have two sides and the angle between them.

Types of Triangles

Isosceles Triangles

1. Two sides are equal.

2. The median drawn from A to BC is also the perpendicular bisector of BC.

3. In the $\triangle ABC$ all the 4 points : the centroid, the orthocentre, the circumcentre and the incentre lie on the altitude drawn from the vertex A to base BC.

Equilateral Triangle

1. All sides and angles (60°) are equal.

2. The median, angle bisector, altitudes, perpendicular bisector of sides are all represented by the same straight lines.

   If the side of an equilateral triangle is a, then

• Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} a^2$

• Altitude to any side $= \dfrac{\sqrt{3}}{2} a$

• Circumradius $= \dfrac{a}{\sqrt{3}}$

- Inradius $= \dfrac{a}{2\sqrt{3}}$

Phythagorus Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$AC^2 = AB^2 + BC^2$$
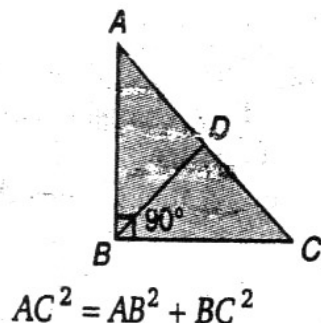
The median BD to hypotenuse $= \dfrac{1}{2} \times$ hypotenuse = circumradius

Area of a Triangle

1. The area of $\Delta = \dfrac{1}{2} \times \text{Base} \times \text{height}$

2. Area of $\Delta = \dfrac{1}{2}$ ab sin θ, where θ is the angle between the sides a and b.

3. Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where $\dfrac{a+b+c}{2}$

where a, b and c are the sides of a triangle.

Median and Centroid

- Line joinint any vertex to the mid point of the opposite side is call the median.
- Median divides the triangle into two equal halves (area-wise)
- Apollonius theorenm

$$2 \times (\text{median})^2 + 2 \times \left(\dfrac{1}{2} \times \text{third side} \right)^2$$

= sum of squares of other two sides.
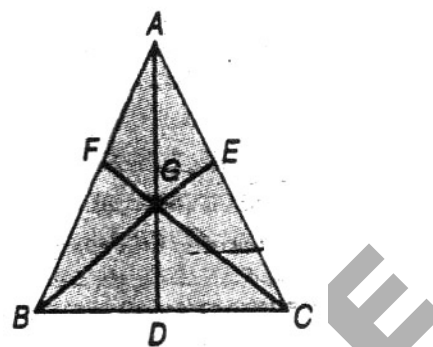
In Δ ABC, $2 \times (AD)^2 + 2 \times \left(\dfrac{BC}{2} \right)^2 = AB^2 + AC^2$

The point where the three medians meet is called the centroid.

it divides the median into two parts in the ratio 2 : 1

Perpendicular bisector and Circumcentre: Perpendicular bisector to any side is the line that is perpendicular to that side and is passing through its mid point.
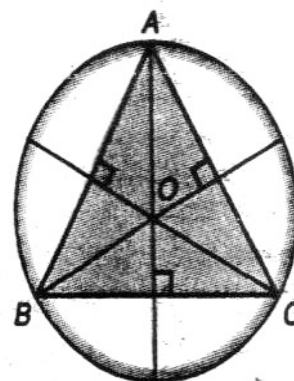
Circumcentre is the point where the three perpendicular bisectors of the triangle meet. Angle blsector and Incentre.

Angular bisector is a line that divides the angles into two equal parts.
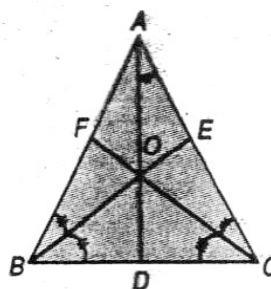
The point where the three angular

bisectors of a tringle meet is the incentre.

Angle bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC} \text{ and } AB \times AC - BD \times DC = AD^2$$

Altitude and Orthocentre

The perpendicular drawn from the vertices to the opposit, sides are called altitudes.
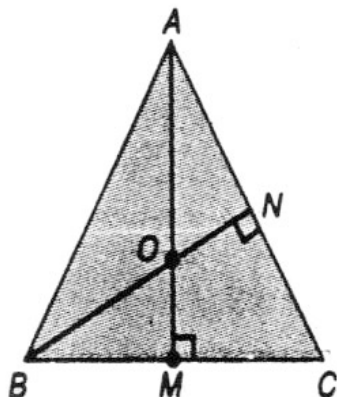
The point where the altitudes to all the three sides meet is called the orthocentre.

Basic Proportionality theorem

Any line parallel to one side of a triangle divides the other two sides proportionally.
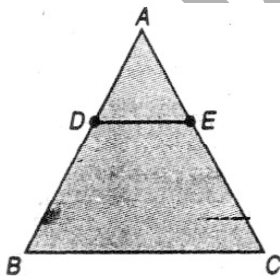
Hence if DE ‖ BC, then

$$\frac{AD}{BD} = \frac{AE}{EC}, \frac{AB}{AD} = \frac{AC}{AE}, \frac{AD}{DE} = \frac{AB}{BC}, ...$$

$$\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$$
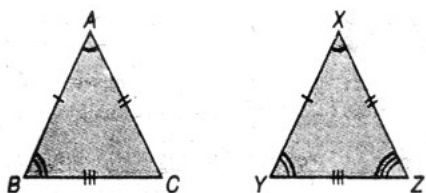
Hence, $\triangle ABC$ and $\triangle XYZ$ are Congruent or $\triangle ABC = \triangle XYZ$

SAS Property (Side Angle-side)

If AB=XY $\angle$A=$\angle$x, AC=XZ

Hence, $\triangle ABC$ and $\triangle XYZ$ are congruent or $\triangle ABC = \triangle XYZ$

AAS Property (Angle-Angle-Side)

If $\angle B = \angle Y$, $\angle C = \angle Z$, AC = XZ

Hence, $\triangle$ ABC and $\triangle$ XYZ are congruent or $\triangle ABC \cong \triangle XYZ$

$\therefore$ $\triangle ABC \cong \triangle XYZ$

SSS Property (Side-Side-Side)

If AB = XY, AC=XZ, BC=YZ

$\therefore$ $\triangle ABC \cong \triangle XYZ$

Similarity of Triangles

Two triangles are similar if

(a) the angle of one Tringle respectively equal to the angle of other.

(b) the corresponding sides are proportional.

properties of similar triangles

(a) Ratio of side = Ratio of height = Ratio of median = Ratio of angle bisector = Ratio of inradius = Ratio of circumradius.

(b) Ratio of Area = Ratio of squares of corresponding sides and height.

ie, if $\triangle ABC$ and $\triangle PQR$ are similar where AD and PS are medians, then

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \frac{AD^2}{PS^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

Polygons

A plane figure enclosed by three or more non-collinear points joined by line segment is called polygon.

Properties of an 'n' sided polygon

1. Sum of interior angles

$$= (2n - 4)\frac{\pi}{2} = (n-2)\pi \text{ radian}$$

$$= (n-2) \times 180^0$$

2. Sum of exterior angles = 360° (always)

3. Number of diagonals $= \dfrac{n(n-3)}{2}$

4. Number of sides $= \dfrac{360^0}{\text{exterior angle}}$

5. Exterior angle $= \dfrac{360^0}{n}$ (in regular polygon)

6. Interior angle + Exterior angle = 180° (always)

7. Area of polygon $= \dfrac{ns^2}{4} \times \cot\left(\dfrac{180^0}{n}\right)$

where s = side length and n = number of side

8. Perimeter = n × s

Quadrilaterals

Quadrilaterals are figures enclosed by four straight lines.
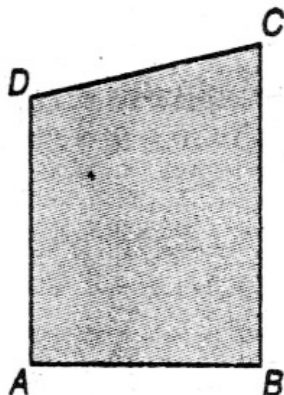
Properties

- Sum of four interior angles = 360°
- By joining the mid points of adjacent sides of a quadrilateral we obtain parallelogram
- If a quadrilateral is circumscribed about a circle, then the sum of opposite sides are always equal.

Types of quadrilateral

(i) **Square:** Its all sides and angles are equal.

(a) Diagonals bisect each other at 90⁰ and are equal in lengths.

(b) When circumscribed about a circle, side of square = diameter

(c) Diagonal $= \sqrt{2} \times$ side

(d) Area = (side)²
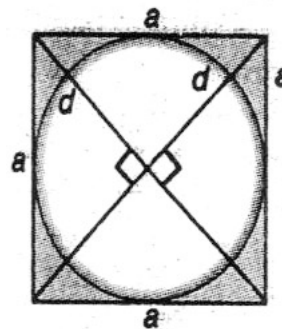
(ii) **Rectangle :** Opposite-sides are-equal and every angle is right angle.

(a) Diagonals bisect each other and are equal in length.

(b) Diameter of circumscribed circle = diagonal of the rectangle.
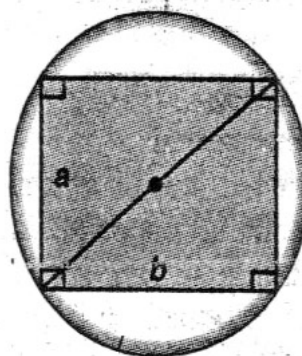
(c) Area = length × breadth and perimeter

= 2(a + b)

(d) Diagonal = $\sqrt{a^2 + b^2}$

(iii) Parallelogram : Opposite sides are parallel and equal.
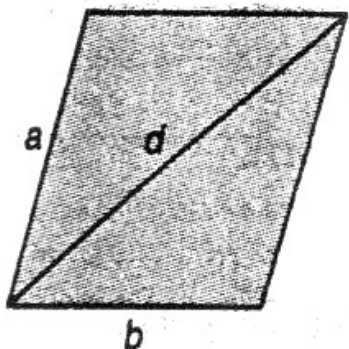
(a) Diagonals bisect each other (the diagonals need not to be equal in length and do not necessarily bisect at right angle).

(b) A parallelogram that is inscribed- in a circle is a rectangle.

(c) When circumscribed about a circle, it becomes a rhombus.

(d) Area=any side × height of parallelogram

(e) Sum of any two adjacent angles = 180°

(f) Each diagonal divided it into two congruent triangles.

(g) Diagonals need not bisect angles at vertices.

(iv) Rhombus: A parallelogram , whose all sides are equal.

(a) Diagonals bisect each other at 90° and not equal except in a square.

(b) Area = $\dfrac{1}{2} \cdot d_1 \cdot d_2$, where d_1 and d_2 are the diagonals of rhombus.

(v) Trapezium: A trapezium is a quadrilateral with a pair of side parallel.
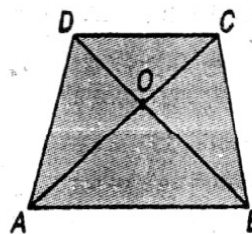
(a) Area = $\dfrac{1}{2}$ × Sum of parallel sides × h

    = $\dfrac{1}{2}(AB + CD) \times h$
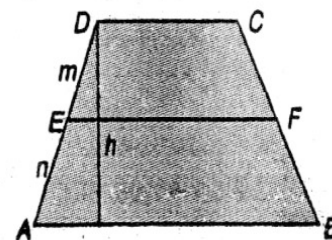
(b) Median=EF=(m × AB+n × DC)/ DC

(c) The line joining the mid points of the oblique side is half the sum of parallel sides.

(d) In trapezium ABCD

$AC^2 + BD^2 = BC^2 + AD^2 + 2AB. CD$

(vi) Cyclic quadrilateral

(a) The four vertices lie on a circle.

(b) If one pair of opposite sides are equal, diagonals are equal.

(c) Sum of product of opposite sides = Product of diagonals.

(d) Opposite angles are supplementary to each other ie, $\angle x + \angle y = 180°$
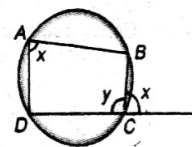
(e) Area of cyclic quadrilateral

= $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$

Where $s = \dfrac{a+b+c+d}{2}$, where

a, b, c and d are its four sides.

CIRCLE

Points Related to Centre of the Circle

1. Only one circle can pass through three given points.

2. Circles with same centre ate concentric circles.
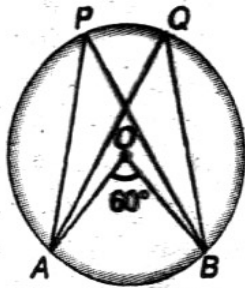
3. When two circles touch, their centres and the point contact are collinear.

4. If two circles touch externally, distance between centres = Sum of radii.

5. If two circles touch internally, distance between centres = Difference of radii.

6. Angle at the centre subtended by an arc = Twice angle subtended by the arc at any point on the remainin part of the circumference.

$$\therefore \quad \angle APB = \frac{1}{2}\angle AOB = 30^0$$

7. Angle in a semicircle is a right angle.

Properties related to chord

1. Perpendicular from centre to a chord bisects the chord.

2. Equal chords of a circle are equidistant from the centre.

3. If two chords AB and CD intersect externally at P, then

$PA \times PB = PC \times PD$
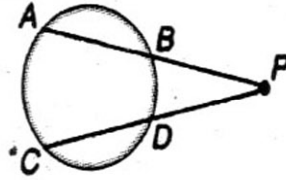
4. If two chords AB and CD intersect internally at P, then

$PAxPB=PCxPD$
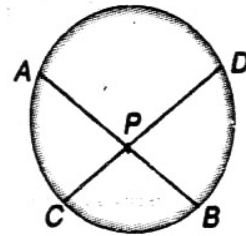
Properties related to tangent

1. Tangent is perpendicular to a radius.
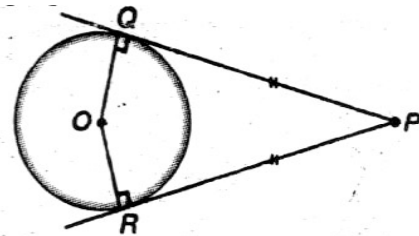
2. The two tangents that can be drawn from an external point are equal.

3. If PAB is a secant and PT is a tangent, then $PA \times PB = PT^2$
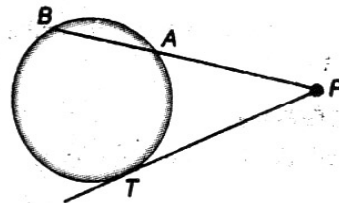
Alternate Segment theorem

If BAC is the tangent at A to a circle and if AD is any chord
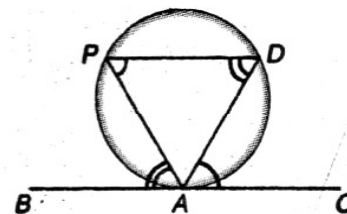
then $\angle DAC = \angle APD$ or $\angle PAB = \angle PDA$

(angles in alternate segment)

Example 1 In the given figure, $\angle MNO$ = 20° and $\angle NMP = 60°$

then find the $\angle MPT$.
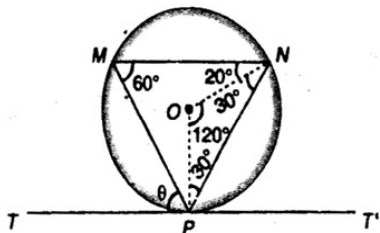
Solution: $\angle PON = 2 \angle NMP = 2 \times 60^0 = 120^0$

$\angle OPN = \angle ONP = 30^0$ (angles opposite to equal sides are equal).

So, $\angle MNP = 50^0 = \angle MPT = 50^0$

Example 2 In the given figure, $\angle AOB = 120^0$, then find the angle $\angle BDE$.
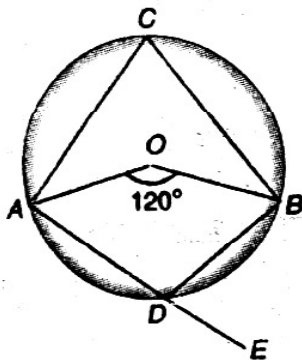
Solution: $\angle ACB = = \dfrac{120^0}{2} = 60^0$

and $\angle BDE = \angle ACB = 60°$

(by property of cyclic quadrilateral)

Example 3 A circle with centre 0 has AB and CD two parallel chords and M and N are mid points of these chords respectively. If AB = 4cm, CD =10 cm and MN = 3crn, then find the radius of the circle.
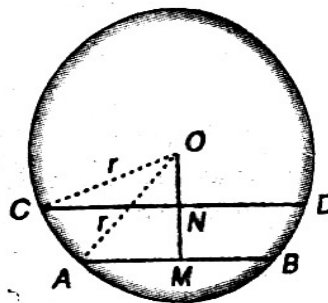
Solution: AB = 4 cm, CD =10 cm, MN = 3cm.

Let $\qquad$ ON = x

$\therefore \quad \Delta OCIV, r^2 = x^2 + 25 \qquad$ ...(i)

In $\Delta OAM, r^2 = (x + 3)^2 + 4$ ...(ii)

From Eqs. (i) and (ii),

$x^2 + 25 = (x + 3)^2 + 4$

$\Rightarrow \quad x = \dfrac{12}{6} = 2cm$

$\therefore \qquad\qquad r^2 = 4 + 25 = 29$

$\therefore \qquad\qquad r = \sqrt{29}$ cm

Example 4 Determine the length of the tangent to a circle of radius 5 cm from a point at a distance of 13 cm from the centre of the circle.
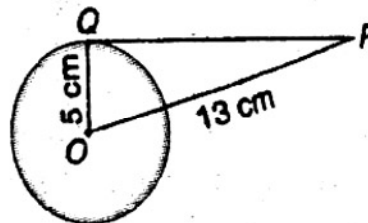
Solution: PQ is a tangent from P which also perpendicular to OQ.

Hence, $PQ^2 = (13)^2 - (5)^2$

$= 169 - 25 = 144$

$\therefore \quad PQ = \sqrt{144} = 12cm$.

Example 5 In the figure $\angle AB = 42°$.

Find $\angle CAT$ and $\angle AEC$.
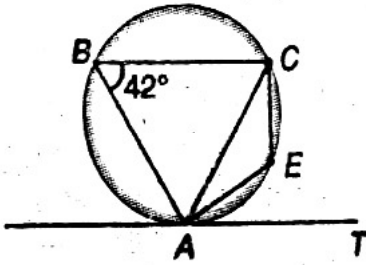
Solution: AT is a tangent at A and AC is a chord. $\angle ABC$ is an angle in the alternate segment of $\angle CAT$
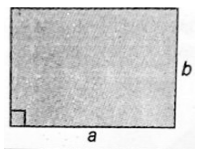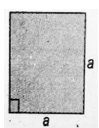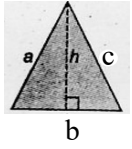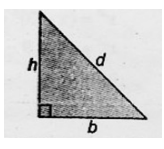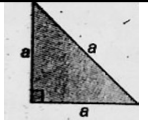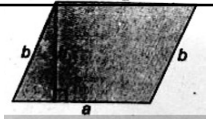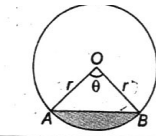
∴ $\angle CAT = 42°$, $\angle AEC + \angle ABC = 180°$
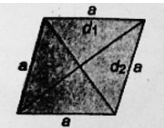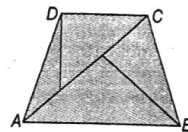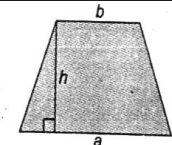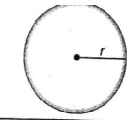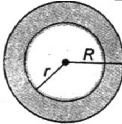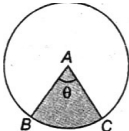
(opposite angles of a cyclic quadrilateral)

∴ $\angle AEC = 180^0 - \angle ABC = 180^0 - 42^0$
$= 138^0$
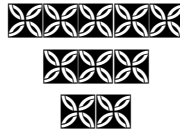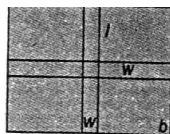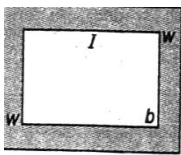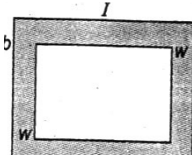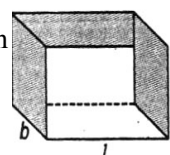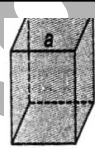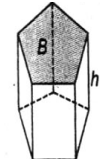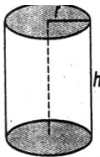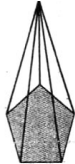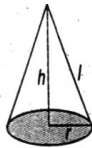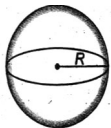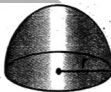
Part - I : Plane Figures

Name	Figure	Perimeter	Area	Nomenclature
Rectangle		2 (a+b)	ab	a = Length b = Breadth
Square		4a	a^2	a= side
Triangle		a+b+c = 2s	1. $\frac{1}{2}b \times h$ 2. $\sqrt{s(s-a)(s-b)(s-c)}$	b is the base and h is the altifude a, b, c are three sides of Δ s is the semiperimeter
Right triangle		b+h+d	$\frac{1}{2}bh$	d (hypotenuse) = $\sqrt{b^2 + h^2}$
Equilateral triangel		3a	1. $\frac{1}{2}ah$ 2. $\frac{\sqrt{3}}{4}a^2$	a = side h = Altitude $= \frac{\sqrt{3}}{2}a$
Isosceles right triangle		2a+d	$\frac{1}{2}a^2$	d (hypotenuse) = $a\sqrt{2}$, a= Each of equal sides.
Parallelogram		2 (a+b)	ah	a=Side b= Side adjacent to a h= Distance between the parallel sides.

Name	Figure	Perimeter	Area	Nomenclature
Rhombus		4a	$\frac{1}{2}d_1d_2$	a= Side of rhombus, $d_1 d_2$ are the two diagonals
Quadrilateral		Sum of its four sides	$\frac{1}{2}(AC)(h_1 + h_2)$	AC is one of its diagonals and h1, h2 are the altitudes on AC from D, B respectively
Trapezium		Sum of its four sides	$\frac{1}{2}h(a+b)$	a, b are parallel sides and h is the perpendicular distance between parallel sides.
Circle		Circumference $=2\pi r$	πr^2	r=Radius of the circle $\pi = \frac{22}{7}$ or 3.1416 (approx).
Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$	r=Radius of the circle
Ring (shaded region)		.....	$\pi\left(R^2 - r^2\right)$	R=Outer radius r = Inner radius
Sector of a circle		l + 2r where $1=\left(\frac{\theta}{360^0}\right)\times 2\pi r$	$\left(\frac{\theta}{360^0}\right)\times \eta r^2$	θ = Central angle of the sector r = Radius of the sector | = Lenght of the arc
Segment of a circle		$\left(\frac{\theta}{360^0}\right)\times 2\pi r$ + 2r sin θ/2	Area of segment ACB (Minor	r = Radius r = Radius

Name	Figure		Area/Perimeter	Nomenclature
			segment) $= r^2 \left[\dfrac{\pi\theta}{360^0} - \dfrac{\sin\theta}{2} \right]$	θ=Angle of the related sector AOB.
Pathways running across the middle of a rectangle		2 (b+l) - 4w	A=w (l+b-w)	l= Lenght b= Breadth w= Width of the path
Pathways outside			A=2w (l+b+2w)	
Pathways inside			A=2w (l+b-2w)	

Name	Figure	Lateral/curved	Total Surface area	Volume	Nomenclature
Cuboid		2h (l+b)	2 (lb+bh+lh)	lbh	l = Length b = Breadth h = Height
Cube		$4a^2$	$6a^2$	a^3	a = Edge
Right prism		(Perimeter of base) $\times$ Height	2 (Area of base) + Lateral surface area	Area of base $\times$ Height	----

Name	Figure	Lateral/curved	Total Surface area	Volume	Nomenclature
Right circular cylinder		$2\pi rh$	$2\pi r\,(r+h)$	$\pi r^2 h$	r = Radius of base h = Height of the cylinder
Right pyromid		$\frac{1}{2}\times$ (Perimeter of the base) $\times$ (Slant height)	Area of the base + Lateral surface area	$\frac{1}{3}$ (Area of the base) $\times$ Height	
Right cicular cone		πrl	$\pi r\,(l+r)$	$\frac{1}{3}\,\pi r^2 h$	h = Height r = Radius l = Slant height
Sphere		--	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$	r = Radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$	r = Radius
Spherical shell		--	$4\pi(R^2+r^2)$	$\left(\frac{4}{3}\right)\pi\left(R^3 - r^3\right)$	R = Outer radius r = Inner radius

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Top Class Employment

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